Bayesian Analysis of Economic Growth in Taiwan and Mainland China via Dynamic Production Functions

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No. 2008-E04
2008.10 (Revised 2009.1)

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Abstract

To analyze the dynamic structures of the economic growth of Taiwan and mainland China, we propose a set of models of dynamic production functions that consider total factor productivity (TFP) and output elasticities as time-varying parameters. Bayesian method based on the smoothness priors approach is applied for parameter estimation. A new approach, called the random grouping method, is proposed to overcome difficulties resulting from rapid changes in the TFP trend. According to the estimation results, we find that the growth rates of TFP in Taiwan have been fluctuated at very low levels since the late 1980s. On the other hand, TFP in mainland China has shown a gentle upward tendency since the late 1970s. Consequently it would appear that the reform and open-door policies since 1978 have contributed to TFP in mainland China.

Keywords: economic growth, total factor productivity, dynamic production function, Bayesian estimation, method of random grouping

JEL classification: C11, C22, C51, O40, O57

1 Introduction

Under the circumstances of the advance of globalization and growing international economic interdependence, the trends for growth of nations have come a major concern for many people such as entrepreneurs, and administration policy-makers. Especially, the high growth performance of China (hereinafter called mainland China) has recently received much worldwide attention because it has achieved remarkable economic development since beginning its economic reforms and opening-up policy in 1978. The process of mainland China's growth has often been compared with that of Taiwan (e.g. Chow (2002, 2007)). Taiwan's rapid growth occurred about two decades earlier than that of mainland China. Among Asian newly industrializing economies (NIEs), the Taiwan economy has achieved significant successes.

The sources of economic growth are generally decomposed into capital, labor and total factor productivity (TFP). As it is obviously difficult to perpetually increase capital or labor, many researchers consider that from a long-term perspective an increase in TFP is the most important factor to achieve sustained economic growth. Hence, grasping trends related to TFP is a crucial issue in empirical research of economic growth. In this paper, we propose a new approach based on Bayesian methodology to estimate the trends in TFP for mainland China and Taiwan.

The two methodologies used in most earlier literature on the growth performance of mainland China and Taiwan have been growth accounting and the econometric estimation of production functions or its variations. For example, reports using the growth accounting approach include Young (1995), Borensztein and Ostry (1996), Singh and Trieu (1999), Wu (2002, ch.5), Wang and Yao (2003), Young (2003), Bosworth and Collins (2008). On the other hand, reports based on the econometric approach include Armer and Liu (1993), Chow (1993), Chow and Li (2002), Chow and Lin (2002), Lin (2003, 2004).

Though earlier studies provide interesting results, we believe that there remains scope for additional research in this field. As pointed out by Barro (1999) and Kyo and Noda (2006), one problem with growth accounting is the assumption that social marginal products can be measured by production factor prices. We believe that this is not necessarily the case if social marginal products coincide with production factor prices. The econometric approach of the production functions or its variations does not rely on the above-mentioned assumption. However, a disadvantage of the econometric approach is that it fails to estimate well trends in TFP. As well, in the econometric approach it is usually assumed that the output elasticities of production factors are constant over time. Such output elasticities may, however, vary over time. For this reason we consider a model in which the output elasticities vary over time. The foundation of the approach applied in this paper is the methodologies for Bayesian linear modeling using the smoothness priors approach, developed by Akaike (1980) and Kitagawa and Gersch (1996). In Jiang (1995) these methods are applied to regression models with time-varying coefficients.

The paper is organized as follows. In the next section, we present the basic models.

The third section introduces a set of Bayesian methods for parameter estimation. Results of the estimation and discussions thereof are given in the fourth section. Finally, the fifth section concludes the paper.

2 Model construction

2.1 Background

The structure of our model, introduced below, is related to that of Wan and Yao (2003), but differs from theirs in several respects. These differences are elaborated below. In the following discussion, we use the symbol t as the time variable.

Wang and Yao (2003) used the growth accounting approach to examine the sources of economic growth in China. They specified the aggregate production function as

$$Q_t = A_t K_t^{\theta} (L_t^* H_t)^{\mu}, \tag{1}$$

where Q_t is the real gross domestic product (GDP), K_t is the stock of physical capital, L_t^* is total employment, and H_t is the stock of human capital, measured as the number of average effective years of schooling per person in the 14-65 age group. Hence, $L_t \equiv L_t^* H_t$ is a skill-adjusted measure of labor input. A_t represents the productivity of an economy when all factor inputs are used. For this reason, it is called total factor productivity, or just TFP. θ and μ are output elasticities with respect to physical capital and skill-adjusted labor, respectively.

In this paper, we regard θ and μ as time-varying parameters together with A_t, and consequently, henceforth, are replaced with θ_t and μ_t. In Wang and Yao (2003), a constant returns to scale (CRS), that is, $\theta + \mu = 1$ is assumed. This assumption was also applied for a dynamic production function in Kyo and Noda (2008) to obtain stability of estimation. In this paper we waive the assumption of CRS because it may cause a biased estimation for the time-varying parameter case.

2.2 Models

To analyze economic growth in Taiwan and mainland China, with a few modifications to the model in Eq. (1) we introduce the dynamic production function as

$$Q_t = A_t K_t^{\theta_t} L_t^{\mu_t}. \tag{2}$$

As stated above, one notable feature of the production function in Eq. (2) is that θ_t and μ_t are considered time-varying parameters together with TFP A_t. Under logarithmic transformation, Eq. (2) can be rewritten as

$$q_t = \theta_t k_t + \mu_t l_t + a_t,$$

where $q_t = \ln Q_t$, $k_t = \ln K_t$, $l_t = \ln L_t$, $a_t = \ln A_t$. Thus, we have a statistical model for a set of sample data of size n as follows:

$$q_t = \theta_t k_t + \mu_t l_t + a_t + \epsilon_t \quad (t = 1, 2, \ldots, n), \tag{3}$$

where ϵ_t is assumed to be a Gaussian white noise sequence with $\epsilon_t \sim \mathrm{N}(0,\ \sigma^2)$, where σ^2 is a constant unknown parameter.

Clearly, we can not obtain valuable estimates of the parameters by a method such as the method of least squares. In this paper, we regard θ_t, μ_t and a_t for $t = 1, 2, \ldots, n$ as random variables from a Bayesian perspective, and apply the smoothness priors approach, introduced by Kitagawa and Gersch (1996), in setting up prior distributions. Smoothness priors are introduced to these parameters based on the assumption that they vary smoothly over time. A set of first order stochastic difference equations is used as priors of the time-varying parameters as follows:

$$\theta_t - \theta_{t-1} = \psi_t, \tag{4}$$

$$\mu_t - \mu_{t-1} = \phi_t, \tag{5}$$

$$a_t - a_{t-1} = \nu_t. \tag{6}$$

Here we assume that ψ_t and ϕ_t are Gaussian white noise sequences with $\psi_t \sim \mathrm{N}(0,\ \sigma^2/d_1^2)$ and $\phi_t \sim \mathrm{N}(0,\ \sigma^2/d_1^2)$.

However, empirical knowledge obtained from Kyo and Noda (2008) implies that there may be rapid changes in the trend of a_t, the logarithmic transformation of TFP. Thus, an added relationship is introduced to the sequence ν_t as

$$\nu_t = c_t + \omega_t, \tag{7}$$

where ω_t is a Gaussian white noise sequence with $\omega_t \sim \mathrm{N}(0,\ \sigma^2/d_2^2)$. Here, c_t is considered an unknown parameter for the case that there is a rapid change on a_t and $c_t = 0$ otherwise. The constraint $\sum_{t=1}^{n} c_t = 0$ is introduced here to ensure that $\sum_{t=1}^{n} \mathrm{E}\{\nu_t\} = 0$.

Moreover, in the above models, θ_0, μ_0, a_0, d_1, d_2 and the parameters in the set $\{c_1, c_2, \ldots, c_n\}$ are as parameters in the prior distributions of the random parameters $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_n)^{\mathtt{t}}$, $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)^{\mathtt{t}}$ and $\boldsymbol{a} = (a_1, a_2, \ldots, a_n)^{\mathtt{t}}$; thus they are called hyperparameters in Bayesian modeling. Here, we regard the hyperparameters as unknown constants. Further, we also assume that ϵ_t, ψ_t, ϕ_t and ω_t are independent of each other. By combining Eqs. (3), (4), (5), (6) together with Eq. (7), a set of Bayesian linear models for the parameters $\boldsymbol{\theta}$, $\boldsymbol{\mu}$ and $\boldsymbol{a}$ can be constructed.

3 Parameter estimation

3.1 Basic scheme

Here we give a basic scheme for parameter estimation based on Jiang (1995), which applied the methodology developed by Akaike (1980).

We assume that there are m non-zero elements, denoted by $\{c_1^*, c_2^*, \ldots, c_m^*\}$, in the set $\{c_1, c_2, \ldots, c_n\}$, and denote a vector of the non-zero elements by $\boldsymbol{c}^* = (c_1^*, c_2^*, \ldots, c_m^*)^{\mathtt{t}}$, so we have $m < n$. We treat the elements of $\boldsymbol{c}^*$ as unknown parameters. From Eq. (3), the likelihood of $\boldsymbol{\theta}$, $\boldsymbol{\mu}$, $\boldsymbol{a}$, and σ^2 is given by

$$f(\boldsymbol{y}|\boldsymbol{\theta}, \boldsymbol{\mu}, \boldsymbol{a}, \sigma^2) = \Big(\frac{1}{\sqrt{2\pi\sigma^2}}\Big)^n \exp\Big\{-\frac{1}{2\sigma^2}||\boldsymbol{y} - \boldsymbol{X}_1\boldsymbol{\theta} - \boldsymbol{X}_2\boldsymbol{\mu} - \boldsymbol{a}||^2\Big\},$$

where $\boldsymbol{y} = (q_1, q_2, \ldots, q_n)^{\mathtt{t}}$, $\boldsymbol{X}_1 = \mathrm{diag}(k_1, k_2, \cdots, k_n)$, $\boldsymbol{X}_2 = \mathrm{diag}(l_1, l_2, \cdots, l_n)$, and $||*||$ denotes the Euclidian norm. Furthermore, from the assumption in Eqs. (4), (5), (6) together with Eq. (7) for a set of given values of σ^2 and the hyperparameters, the prior densities for $\boldsymbol{\theta}$, $\boldsymbol{\mu}$, $\boldsymbol{a}$ are respectively given by

$$\begin{aligned}
\pi_1(\boldsymbol{\theta}|\sigma^2, \theta_0, d_1) &= \Big(\frac{d_1}{\sqrt{2\pi\sigma^2}}\Big)^n \exp\Big\{-\frac{d_1^2}{2\sigma^2}||\boldsymbol{D}\boldsymbol{\theta} - \theta_0\boldsymbol{u}||^2\Big\}, \\
\pi_2(\boldsymbol{\mu}|\sigma^2, \mu_0, d_1) &= \Big(\frac{d_1}{\sqrt{2\pi\sigma^2}}\Big)^n \exp\Big\{-\frac{d_1^2}{2\sigma^2}||\boldsymbol{D}\boldsymbol{\mu} - \mu_0\boldsymbol{u}||^2\Big\}, \\
\pi_3(\boldsymbol{a}|\sigma^2, a_0, \boldsymbol{c}^*, d_2) &= \Big(\frac{d_2}{\sqrt{2\pi\sigma^2}}\Big)^n \exp\Big\{-\frac{d_2^2}{2\sigma^2}||\boldsymbol{D}\boldsymbol{a} - a_0\boldsymbol{u} - \boldsymbol{G}\boldsymbol{c}^*||^2\Big\}, \qquad (8)
\end{aligned}$$

where

$$\boldsymbol{D} = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ -1 & 1 & \ddots & & \vdots \\ 0 & -1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}, \quad \boldsymbol{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

and $\boldsymbol{G} = (g_{ij})$ is an $n \times m$ matrix with $g_{ij} = 1$ for $c_j^* = c_i$ and $g_{ij} = 0$ otherwise.

Then, by putting $\boldsymbol{b}^{\mathrm{t}} = (\boldsymbol{\theta}^{\mathrm{t}}, \boldsymbol{\mu}^{\mathrm{t}}, \boldsymbol{a}^{\mathrm{t}})$, we obtain the marginal likelihood of σ^2 and the hyperparameters as

$$\begin{aligned}
&L(\sigma^2, \theta_0, \mu_0, a_0, \boldsymbol{c}^*, d_1, d_2) \\
&\quad = \int f(\boldsymbol{y}|\boldsymbol{\theta}, \boldsymbol{\mu}, \boldsymbol{a}, \sigma^2)\pi_1(\boldsymbol{\theta}|\sigma^2, \theta_0, d_1)\pi_2(\boldsymbol{\mu}|\sigma^2, \mu_0, d_1)\pi_3(\boldsymbol{a}|\sigma^2, a_0, \boldsymbol{c}^*, d_2)d\boldsymbol{\theta}d\boldsymbol{\mu}d\boldsymbol{a} \\
&\quad = \Big(\frac{1}{\sqrt{2\pi\sigma^2}}\Big)^n d_1^{2n} d_2^m \det(\boldsymbol{W}^{\mathrm{t}}\boldsymbol{W})^{-1/2} \exp\Big\{-\frac{1}{2\sigma^2}||\boldsymbol{W}\widehat{\boldsymbol{b}} - \boldsymbol{h}||^2\Big\}, \qquad (9)
\end{aligned}$$

where

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{X}_1 & \boldsymbol{X}_2 & \boldsymbol{I} \\ d_1\boldsymbol{D} & \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{O} & d_1\boldsymbol{D} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & d_2\boldsymbol{D} \end{bmatrix}, \quad \boldsymbol{h} = \begin{bmatrix} \boldsymbol{y} \\ d_1\theta_0\boldsymbol{u} \\ d_1\mu_0\boldsymbol{u} \\ d_2a_0\boldsymbol{u} + d_2\boldsymbol{G}\boldsymbol{c}^* \end{bmatrix}$$

with $\boldsymbol{O}$ being a zero-matrix, and $\widehat{\boldsymbol{b}} = (\boldsymbol{W}^{\mathrm{t}}\boldsymbol{W})^{-1}\boldsymbol{W}^{\mathrm{t}}\boldsymbol{h}$ denoting the mode of a posterior density,

$$\begin{aligned}
&f(\boldsymbol{b}|\boldsymbol{y}; \sigma^2, \theta_0, \mu_0, \boldsymbol{c}^*, d_1, d_2) \\
&\quad = \frac{f(\boldsymbol{y}|\boldsymbol{\theta}, \boldsymbol{\mu}, \boldsymbol{a}, \sigma^2)\pi_1(\boldsymbol{\theta}|\sigma^2, \theta_0, d_1)\pi_2(\boldsymbol{\mu}|\sigma^2, \mu_0, d_1)\pi_3(\boldsymbol{a}|\sigma^2, a_0, \boldsymbol{c}^*, d_2)}{\int f(\boldsymbol{y}|\boldsymbol{\theta}, \boldsymbol{\mu}, \boldsymbol{a}, \sigma^2)\pi_1(\boldsymbol{\theta}|\sigma^2, \theta_0, d_1)\pi_2(\boldsymbol{\mu}|\sigma^2, \mu_0, d_1)\pi_3(\boldsymbol{a}|\sigma^2, a_0, \boldsymbol{c}^*, d_2)d\boldsymbol{\theta}d\boldsymbol{\mu}d\boldsymbol{a}},
\end{aligned}$$

of $\boldsymbol{b}$ given σ^2 and the hyperparameters. Maximizing the likelihood $L(\sigma^2, \theta_0, \mu_0, a_0, \boldsymbol{c}^*, d_1, d_2)$ in Eq. (9) with respect to the hyperparameters, we obtain their estimates $\widehat{\theta}_0$, $\widehat{\mu}_0$, $\widehat{a}_0$, $\widehat{\boldsymbol{c}}^*$, $\widehat{d}_1$ and $\widehat{d}_2$.

Computationally, if the values of d_1 and d_2 are given, $\boldsymbol{b}$ together with θ_0, μ_0, a_0 and $\boldsymbol{c}^*$ can be estimated simultaneously using the least squares method (see Jiang (1995)). That is, if we set

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{X}_1 & \boldsymbol{X}_2 & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ d_1\boldsymbol{D} & \boldsymbol{O} & \boldsymbol{O} & -d_1\boldsymbol{u} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{O} & d_1\boldsymbol{D} & \boldsymbol{O} & \boldsymbol{0} & -d_1\boldsymbol{u} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{O} & \boldsymbol{O} & d_2\boldsymbol{D} & \boldsymbol{0} & \boldsymbol{0} & -d_2\boldsymbol{u} & -d_2\boldsymbol{G} \end{bmatrix}, \quad \boldsymbol{z} = \begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{b} \\ \theta_0 \\ \mu_0 \\ a_0 \\ \boldsymbol{c}^* \end{bmatrix},$$

the estimate, $\widehat{\boldsymbol{\beta}}(d_1, d_2) = (\boldsymbol{V}^{\mathrm{t}}\boldsymbol{V})^{-1}\boldsymbol{V}^{\mathrm{t}}\boldsymbol{z}$, for $\boldsymbol{\beta}$ is obtained. Here, $\boldsymbol{0}$ denotes a vector of zeros. Thus, the partial likelihood of d_1 and d_2 can be obtained by substituting $\widehat{\boldsymbol{\beta}}(d_1, d_2)$ into Eq. (9), and then the log-likelihoods for σ^2, d_1 and d_2 are given as

$$\ell(\sigma^2, d_1, d_2) = -\frac{1}{2}\{n(\ln(2\pi\sigma^2) + \frac{\widehat{\sigma}^2(d_1, d_2)}{\sigma^2} - 2\ln(d_1^2) - \ln(d_2^2)) + \ln(\det(\boldsymbol{W}^{\mathrm{t}}\boldsymbol{W}))\} \quad (10)$$

with

$$\widehat{\sigma}^2(d_1, d_2) = \frac{1}{n}||\boldsymbol{z} - \boldsymbol{V}\widehat{\boldsymbol{\theta}}(d_1, d_2)||^2.$$

So the estimates $\widehat{d}_1$, $\widehat{d}_2$ of d_1 and d_2 can be obtained by maximizing the function $\ell(\sigma^2, d_1, d_2)$ in Eq. (10) with respect to d_1 and d_2 using a quasi-Newton method. Finally, the estimate of $\boldsymbol{\beta}$ is given by $\widehat{\widehat{\boldsymbol{\beta}}} = \widehat{\boldsymbol{\beta}}(\widehat{d}_1, \widehat{d}_2)$. Hence the final estimates for all the parameters are obtained.

It is well-known that σ^2 can be estimated with $\widehat{\sigma}^2(\widehat{d}_1, \widehat{d}_2)$, which is its maximum likelihood estimate, or, equivalently, the least squares estimate. However, for the data analyzed here, the value of $\widehat{\sigma}^2(\widehat{d}_1, \widehat{d}_2)$ may be very small, which may lead to unstable estimates for the parameters when we apply $\widehat{\sigma}^2(\widehat{d}_1, \widehat{d}_2)$ as the estimate of σ^2. In this paper, we give an estimate for σ^2 in a reasonable way.

3.2 Identifying rapid changes in TFP

Based on the above basic scheme for parameter estimation, we set out a procedure for identifying rapid changes on $\{a_1, a_2, \ldots, a_n\}$; that is, we propose a new approach for identifying $\boldsymbol{c}^*$ from $\{c_1, c_2, \ldots, c_n\}$. The newly-proposed approach is called the method of random grouping.

First, for integers r and s that satisfy $3s+r=n$ and $1 \leq r \leq 3$, we consider that there are not changes on the first r initial values of the sequence a_t, so we put $c_1 = c_2 = \cdots = c_r = 0$. Thus, it is only necessary to identify $\boldsymbol{c}^*$ from $\{c_{r+1}, c_{r+2}, \ldots, c_n\}$. In the process of identifying $\boldsymbol{c}^*$, we assume, temporarily, that all of elements in $\{c_{r+1}, c_{r+2}, \ldots, c_n\}$ are unknown parameters. Clearly, the elements in the set $\{c_{r+1}, c_{r+2}, \ldots, c_n\}$ can be divided into s groups of three because the number of the elements is $3s$. We have $\sum_{t=r+1}^{n} c_t = 0$ from the assumption that $\sum_{t=1}^{n} c_t = 0$; that is, the mean of all the elements in $\{c_{r+1}, c_{r+2}, \ldots, c_n\}$ is zero.

For three elements, say $\widetilde{\widetilde{c}}_1$, $\widetilde{\widetilde{c}}_2$ and $\widetilde{\widetilde{c}}_3$, in a group, we consider an orthogonal transformation of $(\widetilde{\widetilde{c}}_1, \widetilde{\widetilde{c}}_2, \widetilde{\widetilde{c}}_3)^{\mathtt{t}}$ as follows:

$$\boldsymbol{H}(\widetilde{\widetilde{c}}_1, \widetilde{\widetilde{c}}_2, \widetilde{\widetilde{c}}_3)^{\mathtt{t}} = (\widetilde{c}_1, \widetilde{c}_2, \widetilde{c}_3)^{\mathtt{t}},$$

where $\boldsymbol{H} = (h_{ij})$ is a 3-dimensional Helmert matrix with the elements $h_{11} = h_{12} = h_{13} = 1/\sqrt{3}$, $h_{21} = -h_{22} = 1/\sqrt{2}$, $h_{23} = 0$, $h_{31} = h_{32} = 1/\sqrt{6}$ and $h_{33} = -2/\sqrt{6}$. The set $\{\widetilde{\widetilde{c}}_1, \widetilde{\widetilde{c}}_2, \widetilde{\widetilde{c}}_3\}$, can be considered a sample drawn from the set $\{c_{r+1}, c_{r+2}, \ldots, c_n\}$. Thus, it can be seen that $\widetilde{c}_1$ is in proportion to the mean of the sample $\{\widetilde{\widetilde{c}}_1, \widetilde{\widetilde{c}}_2, \widetilde{\widetilde{c}}_3\}$, so $\widetilde{c}_1 = 0$ holds with a larger probability. Therefore, we can set $\widetilde{c}_1 = 0$ and treat $\widetilde{c}_2$ and $\widetilde{c}_3$ as parameters. The above consideration is applied as follows.

Let $\boldsymbol{P}$ denote a $3s$-dimensinal permutation matrix and consider an orthogonal transformation of the vector $(c_{r+1}, c_{r+2}, \ldots, c_n)^{\mathtt{t}}$ as follows:

$$\boldsymbol{H}^*\boldsymbol{P}(c_{r+1}, c_{r+2}, \ldots, c_n)^{\mathtt{t}} = (\widetilde{c}_1, \widetilde{c}_2, \ldots, \widetilde{c}_{3s})^{\mathtt{t}} \tag{11}$$

with

$$\boldsymbol{H}^* = \begin{bmatrix} \boldsymbol{H} & \boldsymbol{O} & \cdots & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{H} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{O} \\ \boldsymbol{O} & \cdots & \boldsymbol{O} & \boldsymbol{H} \end{bmatrix}$$

being a $3s$-dimensional orthogonal matrix. Suppose that in the set $\{\widetilde{c}_1, \widetilde{c}_2, \ldots, \widetilde{c}_{3s}\}$ there are s zero elements and $2s$ non-zero elements. We treat the non-zero elements in $\{\widetilde{c}_1, \widetilde{c}_2, \ldots, \widetilde{c}_{3s}\}$ as unknown parameters and express the vector of these unknown parameters by $\widetilde{\boldsymbol{c}}^*$. Thus, Eq. (11) can be rewritten as

$$\boldsymbol{H}^*\boldsymbol{P}(c_{r+1}, c_{r+2}, \ldots, c_n)^{\mathtt{t}} = \boldsymbol{U}\widetilde{\boldsymbol{c}}^*,$$

or equivalently,

$$(c_{r+1}, c_{r+2}, \ldots, c_n)^{\mathtt{t}} = \boldsymbol{P}(\boldsymbol{H}^*)^{\mathtt{t}}\boldsymbol{U}\widetilde{\boldsymbol{c}}^* \tag{12}$$

with $\boldsymbol{U}$ being a properly-designed $3s \times 2s$ matrix.

From Eq. (6) together with Eqs. (7) and (12), we can set up an another prior density for $\boldsymbol{a}$ as follows:

$$\pi_3^*(\boldsymbol{a}|\sigma^2, a_0, \widetilde{\boldsymbol{c}}^*, d_2) = \Big(\frac{d_2}{\sqrt{2\pi\sigma^2}}\Big)^n \exp\Big\{-\frac{d_2^2}{2\sigma^2}||\boldsymbol{D}\boldsymbol{a} - a_0\boldsymbol{u} - \boldsymbol{e}||^2\Big\}, \tag{13}$$

where $\boldsymbol{e}^{\mathtt{t}} = (c_1, c_2, \ldots, c_r, (\boldsymbol{U}\widetilde{\boldsymbol{c}}^*)^{\mathtt{t}}\boldsymbol{H}^*\boldsymbol{P})$ with $c_1 = c_2 = \cdots = c_r = 0$. By replacing the prior density $\pi_3(\boldsymbol{a}|\sigma^2, a_0, \boldsymbol{c}^*, d_2)$ in Eq. (8) with $\pi_3^*(\boldsymbol{a}|\sigma^2, a_0, \widetilde{\boldsymbol{c}}^*, d_2)$ and using the basic scheme in Section 3.1, we can obtain the estimate of $\widetilde{\boldsymbol{c}}^*$ together with other parameters and hyperparameters. Therefore, all the estimates $\{\widehat{\widetilde{c}}_1, \widehat{\widetilde{c}}_2, \ldots, \widehat{\widetilde{c}}_{3s}\}$ for the elements in $\{\widetilde{c}_1, \widetilde{c}_2, \ldots, \widetilde{c}_{3s}\}$ can be obtained. Hence, we can an estimate $(\widehat{c}_{r+1}, \widehat{c}_{r+2}, \ldots, \widehat{c}_n)^{\mathtt{t}}$ for the vector $(c_{r+1}, c_{r+2}, \ldots, c_n)^{\mathtt{t}}$ using

$$(\widehat{c}_{r+1}, \widehat{c}_{r+2}, \ldots, \widehat{c}_n)^{\mathtt{t}} = \boldsymbol{P}\boldsymbol{H}^{*\mathtt{t}}(\widehat{\widetilde{c}}_1, \widehat{\widetilde{c}}_2, \ldots, \widehat{\widetilde{c}}_{3s})^{\mathtt{t}}$$

which is derived from Eq. (11).

Furthermore, we can obtain N sets of estimates for $(c_{r+1}, c_{r+2}, \ldots, c_n)^{\mathtt{t}}$, each using a permutation matrix constructed randomly. Let $(\widehat{c}_{r+1}^{(j)}, \widehat{c}_{r+2}^{(j)}, \ldots, \widehat{c}_n^{(j)})^{\mathtt{t}}$ be the j-th estimate for $(c_{r+1}, c_{r+2}, \ldots, c_n)^{\mathtt{t}}$, we can use

$$\widehat{\widehat{c}}_i = \frac{1}{N}\sum_{j=1}^{N}\widehat{c}_i^{(j)}, \quad (i = r+1, r+2, \ldots, n) \tag{14}$$

as the final estimates for $\{c_{r+1}, c_{r+2}, \ldots, c_n\}$. Then, we select m elements, that have larger absolute values estimated, as unknown parameters, hence the unknown parameters in the set $\{c_{r+1}, c_{r+2}, \ldots, c_n\}$ are identified.

4 Results and discussions

4.1 Results for Taiwan

To estimate the model for Taiwan, we use real GDP (Q_t), the number of hours worked adjusted by using the number of years of schooling of the working population as a skill-adjusted labor (L_t), and physical capital (K_t) from Chow and Lin (2002). Hence, we can regard the skill-adjusted labor (for descriptive purposes, hereinafter called labor) data of Chow and Lin (2002) as a proxy of human capital. The data of Chow and Lin (2002) cover the period 1951-1999.

Chen (2003) divides the stages of postwar economic development of Taiwan into four periods as: the reconstruction period from 1945 to 1952, the self-sustained period from 1953 to 1963, the high-growth period from 1964 to 1973, and the medium growth period after 1974. The reconstruction period has also been called the economic system reformatting period. In this period, agricultural goods were mainly exported. The self-sustained period is marked by import substitution industrialization, which meant switching policy from imports to domestic production. In the high-growth period, an export-oriented industrialization policy is implemented. Trade during this period is characterized by importing raw materials and exporting manufactured products. In the early 1970s, the Taiwan government adopted a heavy and chemical industry policy to promote the advancement of Taiwan's industrial structure, and since the 1980s has developed industrial policy programs focused on the high-tech industry..

Figures 1 and 2 show the estimated trends in the output elasticity with respect to physical capital ($\widehat{\theta_t}$) and the output elasticity with respect to labor ($\widehat{\mu_t}$) in Taiwan, respectively. The estimated trend in the output elasticity with respect to the physical capital varies between 0.613 and 0.616. On the other hand, the estimated trend in the output elasticity with respect to labor varies between 0.414 and 0.416. As well, because the sum of the output elasticities of physical capital and labor exceeds one, the Taiwan economy is regarded as showing increasing returns to scale in the period 1951-1999.

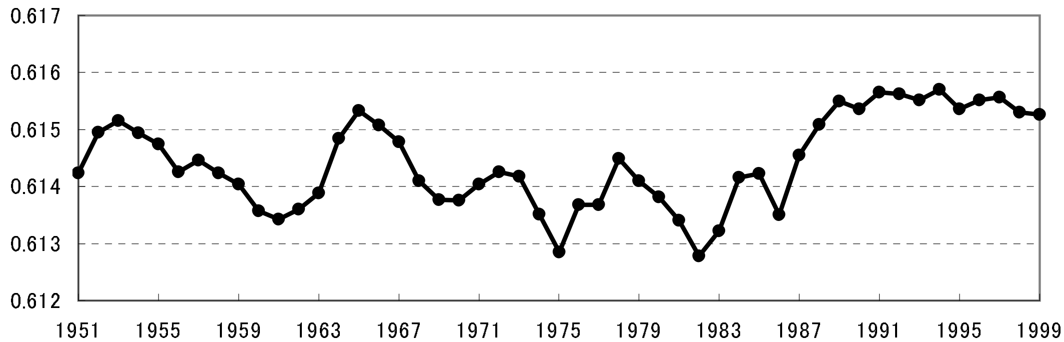

Figure 1: The estimated trend in the output elasticity of physical capital ($\widehat{\theta}_t$) in Taiwan

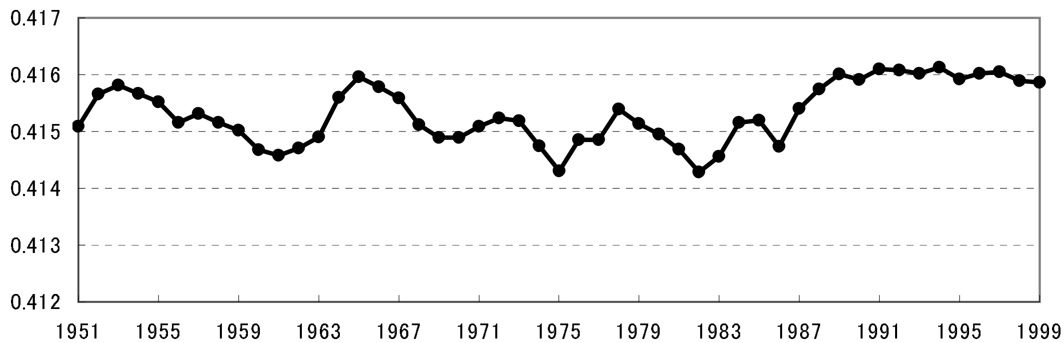

Figure 2: The estimated trend in the output elasticity of labor ($\widehat{\mu}_t$) in Taiwan

From Figures 1 and 2, we find that the estimated trend in both output elasticities with respect to physical capital and labor indicate a similar movement during the period 1951-1999. That is, there are some peaks and troughs from the 1950s to the 1980s. In the 1990s, we show that there is little change in the estimates of both output elasticities. As well, it is found that the estimates of both output elasticities are relatively high compared with those in the previous periods. Therefore, it would appear that the contributions per unit of physical capital and labor to economic growth from the 1990s onward were greater than in previous periods.

Figure 3 shows the trend for estimated TFP ($\widehat{A}_t = \exp(\widehat{a}_t)$) in Taiwan. As is clear from Figure 3, we find that there is a pronounced drop in TFP from the mid-1970s to the mid-1980s. That is, we confirm that there was a precipitous fall in the mid-1970s, and then an accelerated rise in the mid-1980s. Therefore, in the mid-1980's, the TFP had a marked effect on economic growth in the Taiwanese economy. Since the late 1980s, the growth rates of TFP in Taiwan have fluctuated, but at very low levels.

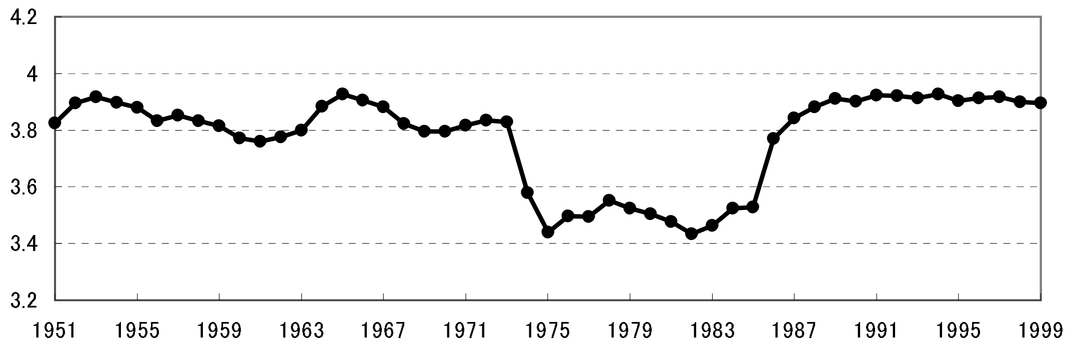

Figure 3: The estimated trend in TFP ($\widehat{A}_t$) in Taiwan

4.2 Results for mainland China

Our sample for mainland China covers a 54-year period (1952-2005). Data for real GDP (Q_t), the stock of physical capital (K_t) and the number of employed persons (L_t^*) are taken from Chow (2008), while data for the average number of schooling years of the working population as a proxy of human capital (H_t) is taken from Minami, Makino and Luo (2008). As in Wang and Yao (2003), we use a skill-adjusted measure of labor, $L_t \equiv L_t^* H_t$, as the data for labor.

First, noting as a background to the social economy of mainland China, political movements may have affected the economic growth in China during the period studied. During the period 1952-2005,there were four main political movements as social campaigns. The first was the Great Leap Forward campaign of 1958-1962. Although Chairman Mao's hope for economic growth was well directed, the Great Leap Forward caused many people to starve to death, and so was a monumental failure. The second was the Cultural Revolution of 1966-1976, which climaxed around 1968. It also caused extensive damage to the Chinese economy, hence it was also a total failure. The third was the reform and opening-up from 1978. This led to the restoration of growth in the mainland Chinese economy. The fourth was the demonstration at Tian'anmen Square in Beijing in the spring of 1989. This caused economic stagnation because for a few years there was a slump in trade and tourism to mainland China.

The estimated trends in output elasticities of physical capital ($\widehat{\theta}_t$) and labor ($\widehat{\mu}_t$) are shown in Figures 4 and 5, respectively. We can see that the trend in output elasticities of physical capital nearly flatten out to a very high level (an average of about 0.887) over the period studied, and the trend in $\widehat{\mu_t}$ parallels that in $\widehat{\theta}_t$ at a very low level (an average of about 0.036). From these Figures, we can see also that these two output elasticities change over time with similar patterns. The output elasticities decrease in the periods 1958-1962 (Great Leap Forward), 1966-1968 (the climax of Cultural Revolution), and 1988-1990 (demonstration at Tian'anmen Square), while they increase from 1978 (beginning of the reform era).

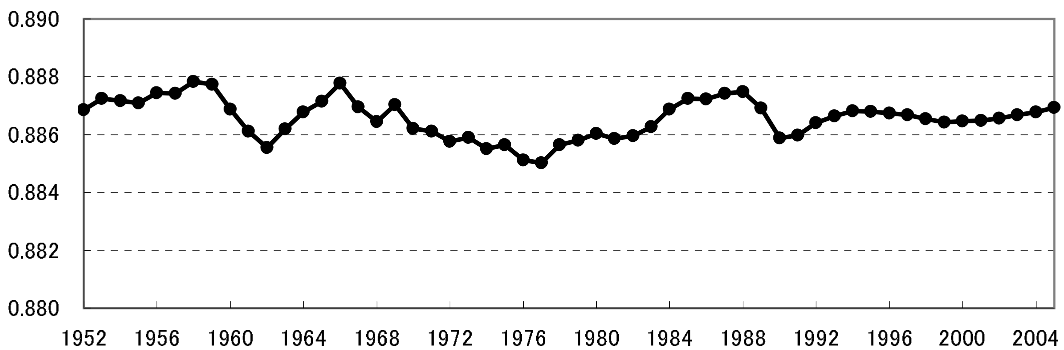

Figure 4: The estimated trend in the output elasticity of physical capital ($\widehat{\theta}_t$) in mainland China

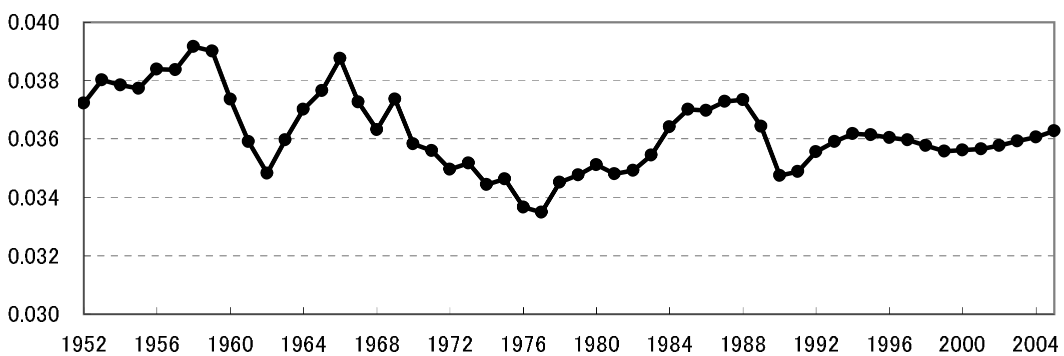

Figure 5: The estimated trend in the output elasticity of labor ($\widehat{\mu}_t$) in mainland China

Moreover, it is found that the estimates of the output elasticities of both physical capital and labor, differ from the values of capital share and labor share computed from national accounts data. In the conventional growth accounting approach, the output elasticities of physical capital and labor is frequently approximated using capital share and labor share. Hence, Our estimation results suggest that researches using the conventional growth accounting approach may lead to the erroneous conclusions.

Figure 6 shows the estimated trend in TFP in the mainland China economy. This result suggests that the trend in TFP may be influenced by the political movements mentioned above. For example, there was a sharp decline around the time at the end of the period 1958-1962 (Great Leap Forward), while there is a full turnaround around the climax of the Proletarian Cultural Revolution. It is interesting that the trend in TFP is mostly downwards during the period 1966-1976 (Cultural Revolution), especially it undergoes a rapid change during the period 1966-1968 (the climax of Cultural Revolution), and then turns slightly upwards during the post-reform period. However, there is a minor trough around 1990, which may be the result of the shock caused by the demonstration at Tian'anmen Square in 1989.

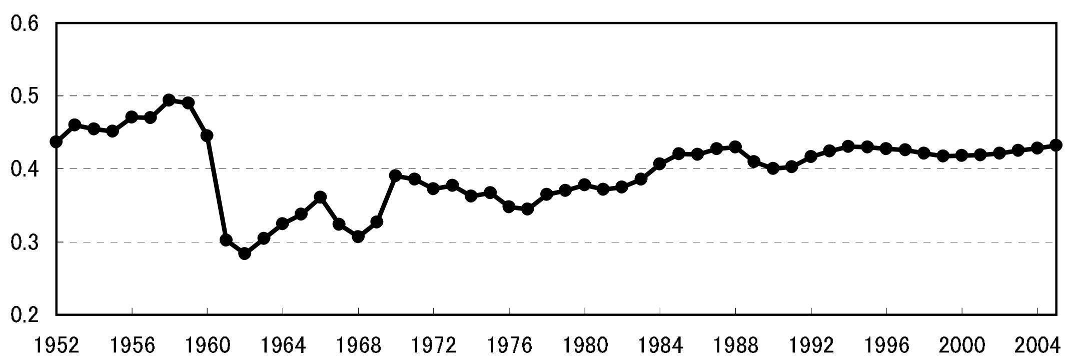

Figure 6: The estimated trend in TFP ($\widehat{A}_t$) in mainland China

Overall, mainland China's TFP has shown a gentle upward tendency since the late 1970s. Consequently it would appear that the reform and open-door policies since 1978 have contributed to mainland China's TFP.

4.3 Discussions

By comparing the above results for Taiwan with those for mainland China, we can see that the estimates of the output elasticities of physical capital and labor are not so different in Taiwan, but they are very different in mainland China. Specifically, the output elasticity of labor is very low in mainland China. As stated in Chow and Lin (2002), the rationale for the low estimate for the output elasticity of labor may be to the result of an abundance of labor in mainland China. Thus, surplus labor may yield almost zero marginal output. On the other hand, Interestingly, the trends in the output elasticities of both physical capital and labor for Taiwan and mainland China show similar patterns. This is useful information for improving modeling in the future studies.

5 Conclusions

To analyze the dynamic structure of the economic growth of Taiwan and mainland China, we have proposed a set of models of dynamic production functions that consider TFP and output elasticities as time-varying parameters. We use the smoothness priors approach for parameter estimation. The new approach, called the random grouping method, was proposed to overcome difficulties resulting from rapid changes in the TFP trend. The results illustrated good performance of the newly-proposed method.

The main results can be summarized as follows. According to the estimation results of Taiwan's aggregate production function, in the period 1951-1999, we found that output elasticity with respect to physical capital changed values from around 0.613 to 0.616, while output elasticity with respect to labor has changed values from around 0.414 to 0.416. We also confirmed that the contributions of per unit of physical capital and labor to economic growth from the 1990s onward were greater than in previous periods. As well, because the sum of output elasticities of physical capital and skill-adjusted labor exceeds one, Taiwan's aggregate production function is regarded as increasing returns to scale during the period 1951-1999. As for TFP in Taiwan, we found that there was a precipitous fall in the mid-1970s, and then an accelerated rise in the mid-1980s. Therefore, in the mid-1980's, the

TFP had a marked effect on economic growth in the Taiwanese economy. Since the late 1980s, Taiwan's TFP growth rates have fluctuated at very low levels.

On the other hand, the estimation results of mainland China's aggregate production function for the period 1952-2005 indicated that output elasticity with respect to physical capital has had values change from around 0.885 to 0.888, while output elasticity with respect to labor has changed in value from around 0.033 to 0.039. Therefore, in China, output elasticity with respect to physical capital was relatively high, whereas output elasticity with respect to labor was very low. Since 2000, both output elasticities have shown a gradual upward tendency. Also, because the sum of output elasticities of physical capital and skill-adjusted labor was below one, mainland China's aggregate production function is regarded as decreasing returns to scale during the period 1952-2005. As for mainland China's TFP trend, it was confirmed that the sharp declines from the late 1950s to the early 1960s and from the mid-1960s to the late 1960s, corresponded to the periods of Great Leap Forward and the climax of Cultural Revolution, respectively. Since the late 1970s, TFP has shown a gentle upward tendency. This suggests that the reform and open-door policies implemented since 1978 have had a positive effect on mainland China's TFP.

Acknowledgments

We gratefully acknowledge the valuable comments of Professor Genshiro Kitagawa at the Institute of Statistical Mathematics, Japan. This study was carried out under the auspices of the ISM Cooperative Research Program (2008-ISM·CRP-2013).

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