An Empirical Study on the Japanese Prefectural Economies Using Bayesian Statistical Models

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Abstract

To examine trends in total factor productivity (TFP) and structural features in Japanese prefectures, this paper presents a new quantitative approach based on a framework of Bayesian statistical models for the regional production functions. When we apply our proposed approach to a regional analysis of the postwar Japanese economy, the following results are obtained. As a regional characteristic in terms of factor elasticities, through the periods of 1955-1973 and 1974-1995, the higher the public capital elasticities, the lower the private capital elasticities in regions. The results also illustrate noteworthy changes in TFP trends before and after the period of first oil crisis. Specifically, the TFP in each prefectural economy shows an upward tendency during the period from the mid-1950s to the early 1970s. Overall, however, it stagnated in the period between the mid-1970s and the mid-1990s.

Keywords: Structural change, Regional growth in Japan, Total factor productivity, Regional production function, Smoothness priors

JEL classification: C11; O18; R11

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1 Introduction

Achieving sustained economic growth is an important issue that has become a policy objective of many countries’ governments. Japan is no exception. Under the recent economic conditions, there have been strong demands in Japan for policy proposals that contribute to the promotion of economic growth.

When considering such policy issues, quantitative analysis of economic growth based on statistical methodology is essential. Empirical studies of economic growth are therefore becoming more important these days. Economic growth analyses done after Solow (1956) often regard capital, labor and total factor productivity (TFP) as the fundamental sources of economic growth. Of these sources, the chronological changes in capital and labor are identifiable from the corresponding data. The trends in TFP, however, cannot be observed directly from the data. In empirical research of economic growth, therefore, the method used to measure TFP behavior constitutes the crucial element to obtain and make useful implications. When we review empirical studies of the Japanese economy, it would appear that the analyses at the regional level are relatively-scarce compared with those at macro and industry levels. Thus, the purpose of this paper is twofold: One is the development of a new quantitative approach for the analysis of regional economic growth. The other is the elucidation of trends in TFP and of characteristic features in Japanese prefectural economies, by the application of our proposed method.

For empirical studies of the Japanese economy using regional data, there are some earlier report; for example, Akino and Hayami (1974) estimated the agricultural production function in Japan for the period 1880-1965 using cross-prefectural data. They also attempted to account for growth in agricultural output with estimates of factor elasticities. As a result, they confirmed that about half of the long-term rate of growth in agricultural output was accounted for by changes in four conventional inputs (i.e., labor, land, capital, and current inputs); one quarter by an increase in the level of education; and another quarter by an increase in public expenditure for agricultural research and extensions. Barro and Sala-i-Martin (1992) compared the process of regional economic growth and absolute convergence across prefectures in Japan and states in the United States, based on a framework of a neoclassical growth model. Absolute convergence means that poor prefectures and states grow faster. Barro and Sala-i-Martin (1992) used two regional data sets, specifically the 47 prefectures of Japan and the 48 states of the United States, to ex-
amine the convergence hypothesis. As a result, they found evidence of convergence in both countries for the periods between 1930 and 1987 for Japan and between 1880 and 1988 for the United States. Asada (1998) investigated the factors’ contributing to output growth in prefectural economies based on a conventional growth accounting approach, using prefectural data for the period 1975-1990. It was confirmed that in many prefectures, the contribution of capital was relatively high; hence, the majority of prefectures corresponded to a capital-driven economy. Fukao and Yue (2000) estimated prefectural production functions using prefectural data for the period 1955-1995, and implemented a growth accounting method with estimates of factor elasticities. Based on their estimation results, they re-examined the convergence hypothesis of Barro and Sala-i-Martin (1992), and then concluded that the convergence hypothesis was not suitable for application to the Japanese economy. Yamano and Ohkawara (2000) investigated the effects of public investment on the regional economies of Japan using prefectural data for the period from 1970 to 1994. Consequently, they showed that the marginal productivity of public capital had recently declined in most depressed regions, whereas productivity in developed regions (e.g., Tokyo, Osaka) had increased slightly. Shioji (2001) constructed a conditional convergence model and then estimated the long run effect of public capital on output per capita using regional data of Japan and the United States; for the period of 1955-1995 for Japan and 1963-1993 for the United States. As well, Shioji (2001) performed a growth accounting exercise using the estimates obtained by econometric analyses. Consequently, the estimation with disaggregated public capital indicated that the effects of public capital on regional output in Japan and the United States were not nearly as dissimilar as they appeared when one considered only aggregate public capital. Utilizing growth accounting, it was confirmed that public capital had made modest contributions to the postwar growth of both Japan and the United States. Kataoka (2005), who focused on the period 1955-2000, examined the relationship between the regional allocation of public capital and regional and national output in the postwar Japanese economy using the prefectural production function approach. The results of his empirical analysis indicated that the Japan’s government allocated more public investment to higher productivity regions prior to the first oil crisis; whereas, after the onset of the first oil crisis, more public investment was allocated to lower productivity regions. Miyara and Fukushige (2008) estimated prefectural production functions with differently aggregated public capital using annual data from 1976 to 1997. Their estimation results indicated that
productive public infrastructure differs between prefectures. From the estimation results, they note that policy-makers should consider prefectural differences in the types of productive public infrastructure.

From the viewpoint of the approaches of empirical study, earlier reports can be roughly classified into two types. The first type uses growth accounting and/or econometric estimations of production functions (e.g., Akino and Hayami, 1974; Asada, 1998; Fukao and Yue, 2000; Yamano and Ohkawara, 2000; Kataoka, 2005; Miyara and Fukushige, 2008). The second type is mainly based on growth regression models derived from economic growth theories (e.g., Barro and Sala-i-Martin, 1992; Shioji, 2001).

However, these approaches might be inadequate. First, growth accounting postulates that under the assumption of a perfectly competitive market for production factors, marginal productivity of each production factor is consistent with the corresponding factor price. Then, it indirectly measures the elasticity of output with respect to production factors using the income shares of such factors. There is the same assumption behind econometric estimations of growth regression models based on neoclassical growth theory. In reality, however, such consistency might not always be achieved, and using indirect data that deviate from true values possibly results in incorrect conclusions.

With regard to the direct estimation of production function models, the condition that the marginal productivity of each production factor coincides with the corresponding factor price need not be required. The traditional regression approach of production functions is unable to appropriately grasp trends in TFP. Because, for convenience, the approach generally assumes that log-TFP is a constant, a linear function of time or a quadratic function of time. Such ad hoc assumptions regarding TFP trends imply that any understanding of the changes in TFP over time is, in effect, excluded from the analysis.

Considering such issues, Kyo and Noda (2006) conducted a statistical analysis of TFP trends in Japan based on a Bayesian approach. Specifically, a Bayesian statistical model of Japanese regional production functions that regards TFP as a time-varying parameter was developed and a smoothness priors approach adopted when estimating TFP trends; thereby, examining TFP changes in the regional economies in Japan. Although Kyo and Noda (2006) took into account the structural heterogeneity among the regions, which means the case in which the elasticity of output with respect to production factors varied across regions, the value of the elastic-
ity was assumed to be constant throughout the period of their analysis. In other words, the analysis did not incorporate the possibility of structural change in regional economies resulting from some kind of exogenous shock. From the perspective of a modeling that is closer to reality, we extend the model of Kyo and Noda (2006) to include structural change. As well, based on the extended model, we make an attempt to estimate TFP trends and analyze the characteristic features of regional economic structures. The approach using the Bayesian statistical model proposed in this study constitutes a new attempt that overcomes some obstacles that have hindered existing approaches as mentioned above. It also supports more detailed analyses than existing approaches do. Therefore, findings that have been overlooked in earlier studies are expected to be uncovered. The findings of this study are expected to contribute to the development of methods of quantitative analysis applicable to regional economic research.

In this study, we examine the possibility of structural change between two sub-periods, 1955-1973 and 1974-1995, using the Japanese Prefectural Database developed in the analysis of Fukao and Yue (2000) that covers annual data between 1955 and 1995. Estimating the elasticity of output with respect to each production factor, the results indicate that interregional variance of physical capital elasticity in the private sector decreased and the variance of physical capital elasticity in the public sector and human capital elasticity increased in both periods. The two sub-periods also showed a strong negative correlation between physical capital elasticity in the private sector and that in the public sector. We also find a clear contrast in TFP trends before and after the first oil crisis. Specifically, in each prefectural economy, TFP increased steadily during the period until the first oil crisis, whereas subsequently it stagnated.

The rest of this paper is organized as follows. In Section 2, we construct statistical models for regional production functions. Section 3 describes the procedure used for parameter estimations in the model. In Section 4, the statistical model and the proposed approach for the estimation is applied by utilizing Japanese prefectural data. Then the estimation results are presented and discussed. Finally, in Section 5, we present a summary of our study and note future research directions.
2 Setup of the Model

2.1 The Regional Production Function Model

Suppose a situation in which a country’s economy is divided into $m$ regions. In the $i$-th ($i = 1, 2, \ldots, m$) region, given the technological state, output $Q_i(t)$ is produced by inputting $K_i(t)$ as the physical capital in the private sector (hereinafter called private capital), $G_i(t)$ as the physical capital in the public sector (hereinafter called public capital), $H_i(t)$ as human capital, and $L_i(t)$ as labor in the period $t$.

In this study, we apply Milbourne, Otto, and Voss’s (2003) specification of macro-production function to our regional analysis. Specifically, the regional production function of the $i$-th region takes the following form.\(^1\)

$$Q_i(t) = K_i(t)^{\alpha_i} G_i(t)^{\beta_i} H_i(t)^{\gamma_i} \left\{ A_i^*(t)L_i(t) \right\}^{1-\alpha_i-\beta_i-\gamma_i} \quad (i = 1, 2, \ldots, m). \quad (1)$$

In Eq. (1), $\alpha_i$, $\beta_i$ and $\gamma_i$ represent unknown parameters. Here it is assumed that $\alpha_i > 0$, $\beta_i > 0$, $\gamma_i > 0$ and $\alpha_i + \beta_i + \gamma_i < 1$. In addition, $A_i^*(t)$ expresses a labor-augmenting measure of productivity.

It should be noted that the productivity indicator, $A_i^*(t)$, is affected not only by the technological level that reflects product and process innovations, but also by diverse factors such as national and regional institutions, central and local government policies, economies of scale, and economies of agglomeration.\(^2\) Such a productivity indicator might be divided broadly between the component reflecting the factors that are unique to each regional economy such as the level of technological infrastructure or the efficiency of regional industries, the situation of industrial agglomeration and natural conditions, and the part that reflects factors that are common among all regions such as widely shared technical knowledge, the extent of the rule of law in the country and the nationwide policies of the central government. From such a perspective, we assume that $A_i^*(t)$ includes two components: The component $C_i$ that does not change over time but which differs among regions; the other component $A(t)$ that is common among the regions but which changes over time. That is, we consider the following relation.

$$A_i^*(t) = C_iA(t).$$

---

\(^1\)Milbourne, Otto, and Voss (2003), however, considered $M$ types of public capital. Our model therefore corresponds to the case of $M = 1$ in their production function.

\(^2\)Hall and Jones (1999) collectively refer to various government policies and systems that affect the activities of economic agents as social infrastructure. Using cross-national analyses, they confirmed that social infrastructure had an important role in long-term economic disparities across countries.
Here we normalize as $A(0) = 1$. Therefore, the relation $A_i(0) = C_i$ holds, and $C_i$ can be interpreted as the initial value of a labor-augmenting measure of productivity of the $i$-th region. When we define $\theta_i \equiv 1 - (\alpha_i + \beta_i + \gamma_i)$, the TFP of the $i$-th region is expressed as $\{C_i A(t)\}^{\theta_i}$. Because $A(t)$, a component of TFP, takes the same value in all regions and relies only on time, we refer to it as a time-varying factor of productivity.

Logarithmic transformation of Eq. (1) is given by

$$y_i(t) = \theta_i \mu_i + \alpha_i x_{1i}(t) + \beta_i x_{2i}(t) + \gamma_i x_{3i}(t) + \theta_i a(t),$$

(2)

where $y_i(t) \equiv \ln\{Q_i(t)/L_i(t)\}$, $x_{1i}(t) \equiv \ln\{K_i(t)/L_i(t)\}$, $x_{2i}(t) \equiv \ln\{G_i(t)/L_i(t)\}$, $x_{3i}(t) \equiv \ln\{H_i(t)/L_i(t)\}$, $a(t) \equiv \ln A(t)$ and $\mu_i \equiv \ln C_i$. Furthermore, we augment Eq. (2) to include a random disturbance:

$$y_i(t) = \theta_i \mu_i + \alpha_i x_{1i}(t) + \beta_i x_{2i}(t) + \gamma_i x_{3i}(t) + \theta_i a(t) + \varepsilon_i(t),$$

(3)

where $\varepsilon_i(t)$ is a disturbance term. Here we assume that if $i \neq j$ or $t_1 \neq t_2$, then $\varepsilon_i(t_1)$ and $\varepsilon_j(t_2)$ are independent of each other and that $\varepsilon_i(t) \sim N(0, \sigma^2)$ $(i = 1, 2, \ldots, m)$. The symbol $\sigma^2$ denotes the variance of the disturbance, which is treated as an unknown parameter.

We refer to the parameters that reflect the economic structure of the $i$-th region, $\alpha_i$, $\beta_i$, $\gamma_i$ and $\theta_i$, as structural parameters. In this paper, we setup a model in which the structural parameters take different values before and after the period at which changes to the economic structure occurred. Consider the annual data of $t = 1, 2, \ldots, n$ for each variable, we assume that a structural change occurred during period $T + 1$, and express the structural parameters from period 1 to period $T$ as $\alpha_i^{(1)}, \beta_i^{(1)}, \gamma_i^{(1)}$ and $\theta_i^{(1)}$; from period $T + 1$ to period $n$ as $\alpha_i^{(2)}, \beta_i^{(2)}, \gamma_i^{(2)}$ and $\theta_i^{(2)}$. Then, Eq. (3) can be rewritten as follows:

$$y_i(t) = \theta_i^{(1)} \mu_i + \alpha_i^{(1)} x_{1i}(t) + \beta_i^{(1)} x_{2i}(t) + \gamma_i^{(1)} x_{3i}(t) + \theta_i^{(1)} a(t) + \varepsilon_i(t),$$

(4)

$$y_i(t) = \theta_i^{(2)} \mu_i + \alpha_i^{(2)} x_{1i}(t) + \beta_i^{(2)} x_{2i}(t) + \gamma_i^{(2)} x_{3i}(t) + \theta_i^{(2)} a(t) + \varepsilon_i(t).$$

In contrast to the model in Eq. (4), which takes into account structural change, the model in Eq. (3) can be interpreted as a model without structural change.
2.2 Parameters and Prior Distribution

Next, we describe the treatment of parameters and the construction of prior distribution. In this paper, the parameters of the model in Eq. (4) are classified into two types; each is estimated using a different method. First, $\alpha_i^{(1)}$, $\beta_i^{(1)}$, $\gamma_i^{(1)}$, $\theta_i^{(1)}$, $\alpha_i^{(2)}$, $\beta_i^{(2)}$, $\gamma_i^{(2)}$, $\theta_i^{(2)}$, $\mu_i$ ($i = 1, 2, \ldots, m$) and $\sigma^2$ are treated as constant parameters and are estimated using the maximum likelihood method. Note that it is constrained by conditions as follows: $\alpha_i^{(j)} > 0; \beta_i^{(j)} > 0; \gamma_i^{(j)} > 0; \theta_i^{(j)} > 0$ and $\alpha_i^{(j)} + \beta_i^{(j)} + \gamma_i^{(j)} + \theta_i^{(j)} = 1$ ($i = 1, 2, \ldots, m; j = 1, 2$). Then, $a(t)$ ($t = 1, 2, \ldots, n$) is treated as a random variable parameter and is estimated using the Bayesian method. Specifically, we introduce the smoothness priors approach to obtain a stable estimate of $a(t)$ ($t = 1, 2, \ldots, n$). Thus, we assume that $a(t)$ changes smoothly over time, and use the following first order stochastic difference equation as a kind of smoothness prior.

$$a(t) - a(t - 1) = \nu(t), \quad \nu(t) \sim \mathcal{N}(0, \sigma^2/\eta^2). \quad (t = 1, 2, \ldots, n) \quad (5)$$

On the basis of the assumption that $A(0) = 1$, the relation $a(0) = \ln A(0) = 0$ holds. In Eq. (5), $\nu(t)$ represents a random disturbance with $\sigma^2/\eta^2$ being an unknown variance. We also assume that $\nu(t_1)$ and $\nu(t_2)$ are independent of each other for $t_1 \neq t_2$ and that $\varepsilon_i(t)$ and $\nu(t)$ are independent of each other for any $i$ and $t$.

3 Estimation Procedures of Parameters

3.1 Matrix-Vector Expression of the Model

The matrix-vector expression of the model in Eq. (4) is as follows:

$$y_i = \mu_i b_i + z_i(S_i) + B_i a + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0_m, \sigma^2 I_n). \quad (i = 1, 2, \ldots, m) \quad (6)$$

In Eq. (6), $y_i = (y_i(1), y_i(2), \ldots, y_i(n))^\top$, $a = (a(1), a(2), \ldots, a(n))^\top$, and $\varepsilon_i = (\varepsilon_i(1), \varepsilon_i(2), \ldots, \varepsilon_i(n))^\top$. As well, $b_i$ and $B_i$ respectively denote an $n$-dimensional vector and a diagonal matrix with the order $n$, which are defined as

$$b_i = \begin{bmatrix} \theta_i^{(1)} \mathbf{1}_T \\ \theta_i^{(2)} \mathbf{1}_{n-T} \end{bmatrix}, \quad B_i = \begin{bmatrix} \theta_i^{(1)} I_T \\ \mathbf{0} \theta_i^{(2)} I_{n-T} \end{bmatrix},$$

where $\mathbf{1}_T$ and $\mathbf{1}_{n-T}$, respectively, denote $T$-dimensional and $n - T$-dimensional vectors whose elements being 1, and $\mathbf{0}$ represent a zero matrix. $S_i = \{\alpha_i^{(1)}, \beta_i^{(1)}, \gamma_i^{(1)}, \theta_i^{(1)}\}$.

\[^3\text{See Kitagawa and Gersch (1996) for details of smoothness priors.}\]
is a set of structural parameters in the \(i\)-th region, and \(z_i(S_i)\) is defined as follows:

\[
z_i(S_i) = \begin{bmatrix}
\alpha_i^{(1)} x_{1i}^{(1)} + \beta_i^{(1)} x_{2i}^{(1)} + \gamma_i^{(1)} x_{3i}^{(1)} \\
\alpha_i^{(1)} x_{1i}^{(2)} + \beta_i^{(1)} x_{2i}^{(2)} + \gamma_i^{(1)} x_{3i}^{(2)} \\
\vdots \\
\alpha_i^{(2)} x_{1i}^{(T)} + \beta_i^{(2)} x_{2i}^{(T)} + \gamma_i^{(2)} x_{3i}^{(T)} \\
\vdots \\
\alpha_i^{(2)} x_{1i}^{(n)} + \beta_i^{(2)} x_{2i}^{(n)} + \gamma_i^{(2)} x_{3i}^{(n)}
\end{bmatrix}.
\]

In addition, \(I_n\) represents the identity matrix of order \(n\), and \(0_n\) denotes an \(n\)-dimensional vector of zeros.

Here the prior of \(a\) in Eq. (5) is expressed as follows:

\[
Da = \nu, \quad \nu \sim N(\nu_n, (\sigma^2/\eta^2)I_n).
\]  

In Eq. (7), \(\nu = (\nu(1), \nu(2), \ldots, \nu(n))^t\) and \(D\) denotes a square matrix of order \(n\), which is defined as

\[
D = \begin{bmatrix}
1 & 0 & \cdots & \cdots & 0 \\
-1 & 1 & \ddots & \ddots & \vdots \\
0 & -1 & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & -1 & 1
\end{bmatrix}.
\]

### 3.2 The Estimation Scheme I

We now introduce a procedure to estimate parameters other than structural ones, based on the given values of the structural parameters. It is as an application of the procedure proposed by Jiang (1995) that is based on the methodology of the Bayesian modeling of Akaike (1980).

First, for \(i = 1, 2, \ldots, m\), the set of estimates of the structural parameters at the estimation stage \(j\) is expressed as \(S_i^{(j)}\), and it is defined as \(S^{(j)} = \{S_1^{(j)}, S_2^{(j)}, \ldots, S_m^{(j)}\}\). If \(S^{(j)}\) is given, then a set of Bayesian linear models for \(a\) is constructed from Eqs. (6) and (7). Here, \(\mu = (\mu_1, \mu_2, \ldots, \mu_m)^t\), \(\sigma^2\) and \(\eta\) are treated as hyperparameters. The posterior distribution of \(a\) is obtained immediately from this model; the mode or average is expressed as \(\hat{a}\). Note that \(\hat{a}\) and \(\hat{\mu}\), which is the estimate of \(\mu\), depending on \(\eta\) and \(S^{(j)}\).
Now, given that $S_{1}(j)$ and the value of constant $\eta$, $\hat{a}$? and $\hat{\mu}$ are simultaneously derived by the following equation.\(^4\)

$$
\begin{bmatrix}
\hat{a}(\eta, S(j)) \\
\hat{\mu}(\eta, S(j))
\end{bmatrix} = (W^{\top}W)^{-1}W^{\top}u,
$$

where $W$ denotes an $(m+1)n \times (n+m)$ matrix and $u$ denotes an $(m+1)n$-dimensional vector, which are defined as

$$W = \begin{bmatrix}
B_1 & b_1 & 0_n & \cdots & 0_n \\
B_2 & 0_n & b_2 & \cdots & \cdots & 0_n \\
\vdots & \vdots & \ddots & \ddots & \ddots & \cdots & 0_n \\
B_m & \cdots & b_m & 0_n & \cdots & \cdots & 0_n \\
\eta D & 0_n & \cdots & \cdots & 0_n
\end{bmatrix},
\quad u = \begin{bmatrix}
y_1 - z_1(S^{(j)}_1) \\
y_2 - z_2(S^{(j)}_2) \\
\vdots \\
y_m - z_m(S^{(j)}_m) \\
0_n
\end{bmatrix}.
$$

Further, the estimate of $\sigma^2$ can be obtained by

$$\hat{\sigma}^2(\eta, S^{(j)}) = \frac{1}{mn} \| u - W \begin{bmatrix}
\hat{a}(\eta, S^{(j)}) \\
\hat{\mu}(\eta, S^{(j)})
\end{bmatrix} \|^2.
$$

Given $S^{(j)}_i$, the log-likelihood of $\eta$ is defined as

$$\ell_1(\eta, S^{(j)}) = -\frac{mn}{2} \left[ \ln \left\{ 2\pi \hat{\sigma}^2(\eta, S^{(j)}) \right\} + 1 \right] - \frac{1}{2} \ln \{ \det(V^{\top}V) \} + \frac{n}{2} \ln(\eta^2). \quad (8)$$

In Eq. (8), $V$ is represented as follows $(m+1)n \times n$ matrix.

$$V = \begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_m \\
\eta D
\end{bmatrix}.
$$

The estimate of $\eta$, that is $\hat{\eta}(S^{(j)})$, is obtained numerically by maximizing $\ell_1(\eta, S^{(j)})$. The estimates of the corresponding $a$, $\mu$ and $\sigma^2$ are given by

$$
\begin{align*}
\hat{a}(S^{(j)}) &= \hat{a}(\hat{\eta}(S^{(j)}), S^{(j)}), \\
\hat{\mu}(S^{(j)}) &= \hat{\mu}(\hat{\eta}(S^{(j)}), S^{(j)}), \\
\hat{\sigma}^2(S^{(j)}) &= \hat{\sigma}^2(\hat{\eta}(S^{(j)}), S^{(j)}).
\end{align*}
$$

\(^4\)See Jiang (1995) for details.
3.3 The Estimation Scheme II

Next, we present a scheme to obtain the estimates for the structural parameters for given values of parameters other than structural ones. Because simultaneously estimating the structural parameters for all regions is extremely difficult, here we consider estimating the structural parameters of each region one by one.

A set of all sample data excluding $y_i$ is expressed as $Y^{(i)}$ ($i = 1, 2, \ldots, m$). As shown in Eqs. (9) and (10), when $S^{(j)}$ is given, $\hat{\eta}(S^{(j)})$, $\hat{\mu}(S^{(j)})$ and $\hat{\sigma}^2(S^{(j)})$ can be obtained. From Eqs. (6) and (7), in addition, the posterior distribution of $a$ based on $Y^{(i)}$ can be obtained as follows.

$$a|Y^{(i)} \sim N\left(\hat{a}^{(i)}, \frac{1}{\hat{\sigma}^2(S^{(j)})}\left((W^{(i)})^t W^{(i)}\right)^{-1}\right),$$

where

$$\hat{a}^{(i)} = \left((W^{(i)})^t W^{(i)}\right)^{-1}(W^{(i)})^t u^{(i)}.$$ 

In Eqs. (11) and (12), $W^{(i)}$ and $u^{(i)}$, respectively, stand for an $mn \times n$ matrix and an $mn$-dimensional vector. Specifically, $W^{(1)}, W^{(2)}, \ldots, W^{(m)}, u^{(1)}, u^{(2)}, \ldots, u^{(m)}$ are defined as

$$W^{(1)} = \begin{bmatrix} B_2 \\ B_3 \\ \vdots \\ B_m \\ \hat{\eta}(S^{(j)}) D \end{bmatrix}, \quad W^{(2)} = \begin{bmatrix} B_1 \\ B_3 \\ \vdots \\ B_m \\ \hat{\eta}(S^{(j)}) D \end{bmatrix}, \quad \ldots, \quad W^{(m)} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_{m-1} \\ \hat{\eta}(S^{(j)}) D \end{bmatrix}.$$  

$$u^{(1)} = \begin{bmatrix} y_2 - \hat{\mu}_2(S^{(j)}) b_2 - z_2(S^{(j)}) \\ y_3 - \hat{\mu}_3(S^{(j)}) b_3 - z_3(S^{(j)}) \\ \vdots \\ y_m - \hat{\mu}_m(S^{(j)}) b_m - z_m(S^{(j)}) \\ 0_n \end{bmatrix}, \quad u^{(2)} = \begin{bmatrix} y_1 - \hat{\mu}_1(S^{(j)}) b_1 - z_1(S^{(j)}) \\ y_3 - \hat{\mu}_3(S^{(j)}) b_3 - z_3(S^{(j)}) \\ \vdots \\ y_m - \hat{\mu}_m(S^{(j)}) b_m - z_m(S^{(j)}) \\ 0_n \end{bmatrix},$$

$$\ldots, u^{(m)} = \begin{bmatrix} y_1 - \hat{\mu}_1(S^{(j)}) b_1 - z_1(S^{(j)}) \\ y_2 - \hat{\mu}_2(S^{(j)}) b_2 - z_2(S^{(j)}) \\ \vdots \\ y_{m-1} - \hat{\mu}_{m-1}(S^{(j)}) b_{m-1} - z_{m-1}(S^{(j)}) \\ 0_n \end{bmatrix}.$$  

Here $\hat{\mu}_i(S^{(j)})$ ($i = 1, 2, \ldots, m$) denotes the $i$-th element of $\hat{\mu}(S^{(j)})$. 

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We now undertake Householder transformation of the $mn \times (n + 1)$ matrix \( [W^{(i)} \ u^{(i)}] \) by using an appropriate $mn \times mn$ orthogonal matrix $H$ as follows:

\[
H \left[ \begin{array}{c} W^{(i)} \\ u^{(i)} \end{array} \right] = \left[ \begin{array}{c} \tilde{W}^{(i)} \\ \tilde{u}^{(i)} \\ O \end{array} \right],
\]

where $\tilde{W}^{(i)}$ represents an upper triangular matrix of order $n$, $\tilde{u}^{(i)}$ denotes an $n$-dimensional vector, and $v^{(i)}$ is an appropriate vector. Based on this transformation, Eqs. (11) and (12), respectively, can be rewritten as follows:

\[
a|Y^{(i)} \sim N\left(\hat{a}^{(i)}, \frac{1}{\hat{\sigma}^2(S^{(j)})}(\tilde{W}^{(i)})^t\tilde{W}^{(i)})^{-1}\right),
\]

\[
\hat{a}^{(i)} = (\tilde{W}^{(i)})^{-1}\tilde{u}^{(i)}.
\]

Let $f^{(i)}(a|Y^{(i)}, S^{(j)})$ be the posterior distribution density function of $a$, which is defined by Eq. (13), and let $f_i(y_i|S_i, a)$ be the likelihood function of $S_i$ and $a$ derived from the model in Eq. (6). Then, the marginal likelihood of $S_i$ is given by

\[
f(y_i|S_i, Y^{(i)}, S^{(j)}) = \int f_i(y_i|S_i, a)f^{(i)}(a|Y^{(i)}, S^{(j)})da.
\]

Consequently, the log-likelihood of $S_i$ is given by

\[
\ell_2(S_i) = \ln \left\{ f(y_i|S_i, Y^{(i)}, S^{(j)}) \right\} = -\frac{n}{2} \ln \left\{ 2\pi \hat{\sigma}^2(S^{(j)}) \right\} - \frac{||u_i - W_i\hat{a}_i||^2}{2\hat{\sigma}^2(S^{(j)})} - \frac{1}{2} \ln \left\{ \det(W_i^tW_i) \right\} + \frac{1}{2} \ln \left\{ \left( \det(\tilde{W}^{(i)}) \right)^2 \right\},
\]

where $\hat{a}_i$ is defined as

\[
\hat{a}_i = (W_i^tW_i)^{-1}W_i^t u_i.
\]

In addition, $W_i$ and $u_i$ represent $2n \times n$ matrix and $2n$-dimensional vector, which are, respectively, defined as

\[
W_i = \left[ \begin{array}{c} B_i \\ \tilde{W}^{(i)} \end{array} \right], \quad u_i = \left[ \begin{array}{c} y_i - \tilde{\mu}_i(S^{(j)})b_i - z_i(S^{(j)}) \\ \tilde{u}^{(i)} \end{array} \right].
\]

Then, at the $(j + 1)$-th estimation stage, the structural parameters in the set $S_i$ can be estimated numerically by maximizing $\ell_2(S_i)$ for $i = 1, 2, \ldots, m$. At this
stage, corresponding to the definition of $S_i$, the set of the estimates for the structural parameters is described as

$$S_i^{(j+1)} = \{ (\alpha_i^{(1)}(j+1), (\beta_i^{(1)}(j+1), (\gamma_i^{(1)}(j+1), (\theta_i^{(1)}(j+1), \\
(\alpha_i^{(2)}(j+1), (\beta_i^{(2)}(j+1), (\gamma_i^{(2)}(j+1), (\theta_i^{(2)}(j+1) \}.$$  

Note that the above maximization of the log-likelihood is realized under conditions of constraint on the structural parameters mentioned in Subsection 2.1. It should also be noted that the above process of parameter estimation is considered for the model in Eq. (4), thus the process of parameter estimation for the model in Eq. (3) can be considered similarly but in a more simple way.

3.4 Algorithm for Parameter Estimation

Based on the parameter estimation schemes I and II mentioned above, an algorithm for parameter estimation can be summarized as follows:

Step 1  
Give a set of appropriate initial values for the structural parameters. Set up, for instance, $(\alpha_i^{(1)}(0)) = 0.25$, $(\beta_i^{(1)}(0)) = 0.25$, $(\gamma_i^{(1)}(0)) = 0.25$, $(\theta_i^{(1)}(0)) = 0.25$, $(\alpha_i^{(2)}(0)) = 0.25$, $(\beta_i^{(2)}(0)) = 0.25$, $(\gamma_i^{(2)}(0)) = 0.25$, $(\theta_i^{(2)}(0)) = 0.25$ ($i = 1, 2, \ldots, m$). Then, calculate $\widehat{\mu}(S(0))$, $\widehat{\sigma}^2(S(0))$ and $\widehat{\eta}(S(0))$, for given $S(0)$ based on estimation scheme I. Further, calculate $\ell_1(\widehat{\eta}(S(0))) = \ell_1(\widehat{\eta}(S(0)), S(0))$ according to Eq. (8).

Step 2  
Perform the following operations for $j = 1, 2, \cdots$.

(i) For given $S^{(j-1)}$, $\widehat{\mu}(S^{(j-1)})$, $\widehat{\sigma}^2(S^{(j-1)})$ and $\widehat{\eta}(S^{(j-1)})$, obtain $S_i^{(j)}$, a tentative estimate of $S_i$, for $i = 1, 2, \ldots, m$ using estimation scheme II.

(ii) Using estimation scheme I, calculate $\widehat{\mu}(S^{(j)})$, $\widehat{\sigma}^2(S^{(j)})$ and $\widehat{\eta}(S^{(j)})$ for the value of $S^{(j)}$ that has been obtained.

(iii) Calculate the value of $\ell_1(\widehat{\eta}(S^{(j)})) = \ell_1(\widehat{\eta}(S^{(j)}), S^{(j)})$ according to Eq. (8).

(iv) If $\ell_1(\widehat{\eta}(S^{(j)})) > \ell_1(\widehat{\eta}(S^{(j-1)}))$ holds, return to (i) and continue the operations; otherwise, proceed to the next step.

Step 3  
Use $\widehat{S} = S^{(j-1)}$ as the set of estimates of the structural parameters in $S$ and use $\widehat{\mu} = \widehat{\mu}(\widehat{S})$, $\widehat{\sigma}^2 = \widehat{\sigma}^2(\widehat{S})$ and $\widehat{\eta} = \widehat{\eta}(\widehat{S})$, respectively, as the estimates of $\mu$, $\sigma^2$ and $\eta$. In addition, calculate the estimate of $C_i$ according to $\widehat{C}_i = \exp(\widehat{\mu}_i)$ ($i = 1, 2, \ldots, m$).
4 Regional Analysis of the Japanese Economy

4.1 Data

The empirical analysis in this paper is performed using data available from the Japanese Prefectural Database developed in the analysis of Fukao and Yue (2000). Although most annual data in the Japanese Prefectural Database cover the period between 1955 and 1995, the data for some variables for Okinawa are available only for the years after 1972 or 1973. Therefore, this paper analyzes 46 prefectures, and excludes Okinawa.

The corresponding relations among the variables in our model and the data from the Japanese Prefectural Database are as follows: We use the gross prefectural domestic expenditure as $Q_i(t)$ data, private capital stock by prefecture as $K_i(t)$ data, public capital stock (industrial infrastructure) by prefecture as $G_i(t)$ data, human capital by prefecture as $H_i(t)$ data, and the number of domestic employed persons by prefecture as $L_i(t)$ data. The values of the gross prefectural domestic expenditure, private capital stock by prefecture and public capital stock by prefecture are measured in real terms using 1980 as the base year.

The method of developing the data described above is explained in the appendix of Fukao and Yue (2000); however, we here provide supplementary remarks on the human capital data, particularly that in the Japanese Prefectural Database. In conventional economic growth literature, human capital is defined as the stock of knowledge and skills acquired by individuals through school education, on-the-job training (OJT) and off-the-job training (Off-JT), and other means. As might be easily imagined, therefore, the actual measurement of human capital is extremely difficult. A data developer consequently faces issues of the perspective from which the human capital should be expressed numerically, and must establish an indicator.
of some kind to understand workers’ skills and ability levels; thus needing to be creative in data development.

Generally, Fukao and Yue (2000) considered that individual skills and ability levels were reflected in workers’ wages and developed their human capital data based on a wage index that takes into account workers’ educational background and sex. More specifically, \( h_{i,t} \), which is the human capital per worker in prefecture \( i \) in year \( t \), is measured as

\[
h_{i,t} = \frac{\sum_{s=1}^{2} \sum_{k=1}^{4} \Omega_{t,s,k} \cdot E_{i,t,s,k}}{L_{i,t}},
\]

where suffix \( s \) denotes sex that \( s = 1 \) is male and \( s = 2 \) is female; suffix \( k \) represents educational background and \( k = 1, 2, 3, 4 \), respectively, denote completion of junior high school or below, high school, junior college or technical college, and undergraduate or higher courses. With such definitions of \( s \) and \( k \), \( \Omega_{t,s,k} \) represents the wage index in year \( t \) of workers whose sex is \( s \) and educational background is \( k \), \( E_{i,t,s,k} \) represents the number of occupied persons in prefecture \( i \) in year \( t \), whose sex is \( s \) and educational background is \( k \), and \( L_{i,t} \) is the number of employed persons in prefecture \( i \) in year \( t \). For instance, \( \Omega_{t,1,2} \) indicates the wage index in prefecture \( i \) in year \( t \) of male workers who had completed high school education and \( E_{i,t,2,3} \) means the number of occupied women in prefecture \( i \) in year \( t \) who had completed a junior college or technical college curriculum. Here the wage index, \( \Omega_{t,s,k} \), of each year is defined as a result of dividing the wage, \( w_{t,s,k} \), of workers whose sex is \( s \) and educational background is \( k \) by the wage, \( w_{t,1,1} \), of male workers who had completed junior high school or below, that is \( \Omega_{t,s,k} = w_{t,s,k}/w_{t,1,1} \).

### 4.2 Results and Discussion

As a preliminary discussion, we examine the model performance of Eqs. (3) and (4). With regard to the model of Eq. (4), we focus on the first oil crisis in 1973, and consider the case that structural parameters between two sub-periods, called Period I (1955-1973) and Period II (1974-1995), respectively, may have different values. For the values of Akaike’s Information Criterion (AIC), we obtain \(-6991.8\) for the model of Eq. (3) and that \(-7298.6\) for the model of Eq. (4).\(^7\) According to the

\[^7\text{In our model, the AIC value is calculated according to the following equation.}
\]

\[
AIC = -2 \times \ell_1(\hat{\eta}(S^{(j-1)})) + 2 \times \text{the number of free parameters},
\]

where the number of free parameters of a model without the structural change is \(3m+2\); the number of free parameters of a model that takes into account the structural change is \(6m+2\).
minimum AIC method proposed by Akaike (1974), the model with a smaller AIC value is considered better. The model of Eq. (4) that incorporates structural change is therefore regarded as a better model. Hence, we discuss as follows the estimation results from the model of Eq. (4).

Tables 1 and 2 present the estimated values of each production factor elasticity based on the model of Eq. (4) in Japan’s 46 prefectures, excluding Okinawa, for Period I and Period II.

Table 1: Estimates of production factor elasticities during Period I (1955-1973)

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<tr>
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<th>( \hat{\alpha} )</th>
<th>( \hat{\beta} )</th>
<th>( \hat{\gamma} )</th>
</tr>
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<td>Hokkaido</td>
<td>0.50809</td>
<td>0.06739</td>
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<tr>
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<td>Shimane</td>
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<tr>
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From Tables 1 and 2, we find that production factor elasticity takes various values among the prefectures. As well, it is confirmed that such elasticity takes different values through Periods I and II, suggesting the occurrence of structural change. Regarding the variance of the elasticity of each production factor among prefectures, the variance of private capital elasticity was 0.064 in Period I and 0.020
Table 2: Estimates of production factor elasticities during Period II (1974-1995)

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<th>$\hat{\gamma}$</th>
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<td>0.39598</td>
</tr>
<tr>
<td>Nagasaki</td>
<td>0.45787</td>
<td>1.11E-10</td>
<td>0.40882</td>
</tr>
<tr>
<td>Kumamoto</td>
<td>0.26425</td>
<td>0.23233</td>
<td>0.25558</td>
</tr>
<tr>
<td>Oita</td>
<td>0.24378</td>
<td>0.32557</td>
<td>0.17753</td>
</tr>
<tr>
<td>Miyazaki</td>
<td>0.39288</td>
<td>0.07436</td>
<td>0.35808</td>
</tr>
<tr>
<td>Kagoshima</td>
<td>0.38431</td>
<td>0.10351</td>
<td>0.32048</td>
</tr>
</tbody>
</table>

in Period II. While that of public capital elasticity was 0.019 in Period I and 0.026 in Period II, and that of human capital elasticity was 0.008 in Period I and 0.019 in Period II. A comparison of Period I and Period II therefore indicates a decrease in the variance of private capital elasticity and an increase in the variance of human capital and public capital elasticity.

The characteristics of a structural changes in prefectures in terms of production factor elasticity are shown in Figure 1 based on the estimation results presented in Tables 1 and 2.
Figure 1: Characteristics of structural changes in 46 prefectures, excluding Okinawa
In Figure 1, (a), (b) and (c) correspond to private capital elasticity, public capital elasticity, and human capital elasticity, respectively. The positions of the prefectures in which the elasticity of each factor increased from Period I to Period II are shown in yellow and the positions of prefectures in which the elasticity declined are shown in gray. In the dynamics of the structural changes in the prefectural economies viewed by factor elasticity, the structural movement inherent in each prefecture is visible from the graph in Figure 1. The directions of the changes in the elasticity of each factor sorted by color reveal the characteristic that some neighboring prefectures made changes in the same direction, which suggests close economic correlations among these neighboring prefectures.

In an attempt to understand the structural characteristics of regional economies in Japan, we now examine what kind of relevance exists among the elasticity values estimated for each factor. Tables 3 and 4 present the correlation coefficients of the factor elasticity during Period I and Period II, respectively.

Table 3: Correlations among production factor elasticities during Period I (1955-1973)

<table>
<thead>
<tr>
<th></th>
<th>Private capital ((\hat{\alpha}))</th>
<th>Public capital ((\hat{\beta}))</th>
<th>Human capital ((\hat{\gamma}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private capital ((\hat{\alpha}))</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public capital ((\hat{\beta}))</td>
<td>-0.881</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Human capital ((\hat{\gamma}))</td>
<td>-0.338</td>
<td>-0.139</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 4: Correlations among production factor elasticities during Period II (1974-1995)

<table>
<thead>
<tr>
<th></th>
<th>Private capital ((\hat{\alpha}))</th>
<th>Public capital ((\hat{\beta}))</th>
<th>Human capital ((\hat{\gamma}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private capital ((\hat{\alpha}))</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public capital ((\hat{\beta}))</td>
<td>-0.869</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Human capital ((\hat{\gamma}))</td>
<td>0.638</td>
<td>-0.931</td>
<td>1.000</td>
</tr>
</tbody>
</table>

From Tables 3 and 4, a high negative correlation between private capital elasticity and public capital elasticity is shown through both Periods I and II. Regarding the relation between private capital elasticity and human capital elasticity, a negative correlation is found during Period I, although a positive correlation appears during Period II. However, the absolute values of the correlation coefficients are not very large. Regarding the relation between human capital elasticity and public capital elasticity, although Period I shows a low negative correlation, Period II presents a high negative correlation. The existence of the high negative correlation between
private capital elasticity and public capital elasticity throughout the two periods is particularly noteworthy. This, in general, suggests the characteristic of regional economies in Japan that the lower the private capital elasticity in the region, the higher the public capital elasticity.

Next, we compare and study the initial value, $\hat{C}_i$, of the labor-augmenting measure of productivity among prefectures. Figure 2 represents a graph drawn based on the estimation results of the initial values of productivity indicators for 46 prefectures, excluding Okinawa.

![Figure 2: Estimated initial values of the labor-augmenting measure of productivity](image)

Figure 2 shows a prominently high estimated level for Tokyo. The second highest value is the estimate for Osaka, which, however, is less than half the estimated value for Tokyo. The productivity indicator of Tokyo in the mid-1950s, therefore, appears to have already been much higher than those of other prefectures. After Tokyo and Osaka, the estimates for such prefectures as Kanagawa, Kyoto, and Aichi are high. With a few exceptions, prefectures having an ordinance-designated city are generally ranked highly. For a comparison of the estimates for prefectures other than the so-called three major metropolitan areas that have Tokyo, Osaka and Nagoya as their
cores, we can not find any salient differences.

How can such estimation results of the initial values of the productivity indicators be interpreted? For this issue, the detailed analysis made by Fujita and Tabuchi (1997) of the structural background of regional economic growth in Japan after World War II (hereafter WWII) presents some useful suggestions. Following Fujita and Tabuchi (1997), we give a brief historical overview of regional growth in postwar Japan.\(^8\) The regional economies in Japan after WWII generally developed through the following process. In Japan in 1946, immediately after the war, GNP declined to approximately half of its peak before the war; there was an economic boom during the Korean War (1950-1951) and by the mid-1950s GNP had recovered to the pre-war peak level. The condition of the regional economies in Japan around the first half of the 1950s is characterized by the term *Tokyo-Osaka bipolar regional system*. The subsequent period from the mid-1950s to the first half of the 1970s was a time of rapid growth in the Japanese economy. During this period, the major sector in Japan’s manufacturing industry shifted from the light industries to the heavy and chemical industries, with an accompanying conversion from the Tokyo-Osaka bipolar regional system to the *Pacific industrial belt system* (extending from Tokyo to Kitakyushu). Because of the effect of the first oil crisis, leading industries in Japan shifted from the heavy and chemical industries to the high technology and service industries that were more energy-efficient and internationally competitive after the mid-1970s. In other words, the importance of more knowledge-intensive production activities increased and therefore a knowledge-based economy developed. During this transition process, Japan’s regional economic system shifted from the Pacific industrial belt system to the *Tokyo monopolar system*. Since then, a regional system of the sole concentration being in the Tokyo metropolitan area with Tokyo as its core has continued to date.

Year 1955, which is the initial year of the period (1955-1995) that is analyzed in this study, is the period of transition from the Tokyo-Osaka bipolar regional system to the Pacific industrial belt system described earlier. Fujita and Tabuchi (1997) state that the year 1955 marked the point when the rate of net migration to the three major metropolitan areas of Tokyo, Osaka, and Nagoya began to increase sharply; the rate of net migration to the Tokyo area was the highest among the three major cities. When particularly addressing the share of output of the manufacturing in-

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\(^8\)See Falth (2005), for instance, for the history of Japanese economic growth from the Edo period until the postwar recovery period.
dustry in 1955, that of the Tokyo metropolitan area was also the highest, followed by the Osaka and Nagoya areas. While Fujita and Tabuchi (1997) emphasize the agglomeration economies in the Tokyo area after the mid-1970s, comprehensive consideration of the results of their analysis suggests that the agglomeration economies in the Tokyo area in the mid-1950s were already far greater than in other regions. The agglomeration economies in the Osaka and Nagoya metropolitan areas should also have been relatively large, following Tokyo. Therefore, the results of the estimation of the initial values of productivity indicators are most likely to reflect the extent of agglomeration economies. In other words, the estimation results related to the initial values of productivity indicators are considered to be mostly consistent with the extent of the agglomeration economies in each prefecture during the mid-1950s.

Next, we investigate the trends in TFP. In our model, the TFP in the prefectures in each period is determined by $C_i$, $\theta_i$ and $A(t)$. Recall that, based on the assumption, $C_i$ remains constant through all periods and $\theta_i$ takes a constant value in each Period (namely Periods I or II). For this reason, the direction of the changes in prefectoral TFP in each Period is specified by $A(t)$. Then, depending on the differences in the size of $C_i$ and $\theta_i$ of each prefecture, the TFP of each prefecture shifts in parallel. Thus, we can calculate the direction of changes in prefectoral TFP by investigating the movement of the time-varying factor of productivity, that is $A(t)$. In Figures 3 and 4, the vertical axis represents the natural logarithm of the estimated time-varying factor of productivity (i.e., $\hat{a}(t) = \ln \hat{A}(t)$); the horizontal axis represents the years. The estimated values of the time-varying factor of productivity during Periods I and II are shown.

![Figure 3: Changes in the time-varying factor of productivity during Period I (1955-1973)](image-url)
Figure 4: Changes in the time-varying factor of productivity during Period II (1974-1995)

The changes in the time-varying factors of productivity during Periods I and II indicate the following characteristics. First, Figure 3 shows that the time-varying factor of productivity shifts upward through Period I. The movement of the time-varying factor of productivity during Period II observed in Figure 4 implies a generally sluggish trend during this period, albeit with slight increases at the end of the 1970s and at the end of the 1980s. During the 1990s, in particular, a sign of a slow decline in the trends appears. Hence, the changes in the TFP trends of each prefecture can be inferred as follows based on information related to changes in such time-varying factors of productivity. In other words, the TFP of each prefecture showed a trend to increase from the mid-1950s to the first half of the 1970s. After the mid-1970s, however, the trend changed toward a state of stagnation.

Based on the estimation results, we consider the factors underlying such TFP trends. Because TFP is unquestionably simultaneously affected by various factors and because our estimation results alone cannot provide definitive conclusions, the interpretation requires careful consideration. Considering this, we might be able to make a general interpretation as described below. First, the steady increase in TFP during Period I (1955-1973), suggested by Figure 3, might be reflected by technological progress based primarily on technology imported from North America and Europe. As pointed out by Ohkawa and Rosovsky (1973), Peck and Tamura (1976), Wakasugi (1986), Goto (1993) and others, Japanese industries aggressively adopted advanced foreign technologies in the post-WWII era and actively invested in research and development (R&D) in an effort to create improved technology based on that adopted. The R&D activities based on profit motives of firms not only raise the technology levels of these firms, but promoted technological progress of the
macro economy as the technical knowledge with extensive applicability among other new technologies spread to other industries beyond immediate boundaries.

Here we briefly give some background to the technological adoption that occurred in Japan in the post WWII era, according to Wakasugi (1986). During the chaotic period during and after WWII, Japan’s institutions allowed no introduction of technology from abroad. It was not until 1950 that the import of overseas technology began again. Figure 5, depicting the number of approved contracts for importation of foreign technology, represents the combined total of such contracts in the chemical industry, and those dealing with petroleum products, steel and iron, and general, electrical and transport machinery of Japanese firms during the period from 1950 to 1969.

![Figure 5: Numbers of contracts approved for the introduction of foreign technology](image)

Note: The number of contracts includes only ‘class A’ technical agreements having a contractual or payment term of one year or longer.

Source: Data from Wakasugi (1986).

As might be evident from Figure 5, there was no increasing trend in the number of approved contracts for the importation of foreign technology identified during the 1950s. This is attributable to the considerable restrictions imposed on technology import by the then Japanese government. In 1959, approval for the adoption of overseas technology was relaxed. Subsequently in 1968, adoption of technology was completely deregulated. The number of technological adoption cases increased substantially because of this series of policy changes.

The remarkable effect of the government’s approval system for the introduction of overseas technology on the formation of the industrial structure of the Japanese economy is worthy of mention. Wakasugi (1986) states that in 1950, the Ministry
of International Trade and Industry announced 33 technologies for which the adoption was to be encouraged. Such technologies were related mostly to the heavy and chemical industries such as chemicals, steel and machinery. The technology list added to 1950 was also concerned the rationalization and modernization of production processes in the heavy and chemical industries. Although it did not mean that the technologies of sectors not named in the technology list were prohibited from being adopted, the technologies that could be adopted in the heavy and chemical industries are thought to have been biased towards at the time when technological adoption required strict permission from the government. In other words, because the promotion of the heavy and chemical industries was one economic goal of the government, the development of Japan’s heavy and chemical industries was formed according to the government’s intentions for its policies.

The migration of numerous workers from the agricultural sector to the manufacturing sector during this time also constitutes a significant factor. The increase in the labor movement from the agricultural sector, with relatively low TFP, to the manufacturing sector, with higher TFP, increased the efficiency of resource allocation and promoted TFP growth at the macro level.

We now turn to discuss the causes affecting the slowdown of TFP during Period II (1974-1995). As apparent from a comparison of Figures 3 and 4, the movement of the time-varying factor of productivity stagnated at the time of the first oil crisis; negative growth was then experienced during some periods. Thus, after 1974, prefectural TFP also is considered to have entered a phase of stagnation.

How can we interpret the slowdown in TFP growth during Period II (1974-1995)? As argued in Moriguchi (1988), for example, the first oil crisis gave rise to a substantial decline in capacity utilization in energy-intensive industries such as the petrochemical and the, petroleum refining raterated industries. It took a fair amount of time to restore capacity utilization to its functional level. So, the deterioration in energy efficiency led to the obsolescence of some existing production equipment.

In the early 1970s, the transition of the foreign currency exchange system from fixed to floating rates was also a remarkable occurrence. This led a swift appreciation of Japanese yen in terms of US dollars, and exaggerated the response of government and the Central Bank of Japan at that time, such as in the excessive public investment by the government and the excessive increase of the money supply by the Central Bank of Japan. As a result, such policy failures gave rise to high inflation, which the first oil shock promoted.
Further, as pointed in Valdés (2003), from the mid-1970s some Japanese industries became the owners of worldwide leading-edge technology. This means that the pace of imitation-based technological improvement declined. Hence, such situation led to a high requirement for the development of original state-of-art technology in Japan. As is well known, however, creating new technology oneself compared with introduction of imitation technology requires large R&D budgets. As well, it also has a high possibility of failure compared to the introduction of imitation technology. For that reason, it follows that the speed of technological progress would slow.

Thus, there were policy failures and confusions associated with the exogenous shocks of the first oil crisis and the transition to a floating exchange system. In addition, by reaching positions at the technological frontier, the advantages of backwardness were mostly lost. As well, it should be noted that since the 1970s Japan has had a rapidly aging population. For example, Noda (2007) analyzed the relationship between population aging and technological progress using an endogenous innovation model, and found that population aging may have a negative impact on the rate of technological progress at a macroeconomic level. From the viewpoint of Noda’s (2007) model, hence, rapid population aging since the 1970s might have contributed to the decline in the rate of technological progress in Japan. For the early part of Period II (1974-1995), we infer that the slowdown in the TFP growth can be attributed to a mixture of causes, as mentioned above.

Next, we refer the slowdown of TFP growth after the 1990s. In addition to the continuing aging problem, we considered the following causes. In Hayashi and Prescott (2002), for instance, the decline in the TFP trend in the Japanese economy in the 1990s was interpreted as being caused by loan policies for inefficient firms and declining industries. Sakuragawa (2003) more specifically developed the interpretation of Hayashi and Prescott (2002) and convincingly demonstrated, based on various data, that the major cause of the decline in the TFP trends in Japan during the 1990s was the inefficiency of resource allocation. The argument of Sakuragawa (2003) can be summarized as follows: Bad-loan problems surfaced in Japan at the time of the collapse of the bubble economy at the beginning of the 1990s; the facts that no short-term solution was introduced and that the problem persisted for a long time had negative effects on the Japanese economy. For instance, additional loans were provided to bad borrowers and the over-banking phenomenon with excessive deposits promoted the soft budget problem, which preserved a considerable number of inefficient firms, thereby hindering the movement of normal production factors.
from low-productivity industries to high-productivity ones. The inevitable results were to prolong the problem of bad loans and to further increase inefficient resource allocation. The series of such negative effects is a conceivable cause of the sluggish TFP growth during the 1990s. Sakuragawa (2003), therefore, asserts that promptly solving bad loan problems using an appropriate policy mix is important.

What other policies are feasible for an improvement of TFP and therefore promotion of economic growth? One might be industrial policies related to R&D subsidies. As emphasized by endogenous growth theory (see, for example, Grossman and Helpman, 1991; Aghion and Howitt, 1998; Barro and Sala-i-Martin, 2004), technical knowledge and information accumulated in R&D activities contribute to the technological advancement of the macro economy through positive externality; namely, spillover effects. Nakamura (2003) analyzed the external effects of technical knowledge in Japan’s manufacturing industry between 1968 and 1996 using econometric analysis and discovered that the spillover of technical knowledge had been a major determinant of TFP growth, even after the 1990s. According to our analysis, however, TFP growth during the 1990s was stagnant. It seems very apparent that TFP reflects the effects of multiple factors and that the slow TFP growth in the 1990s must have been strongly affected by problems related to the bad loan issue described earlier. Nevertheless, the possibility that the conventional policy of R&D subsidies was inefficient is also suggested. As pointed out also by Nakamura (2003), that depending on the field, spillover of technical knowledge alone creates diverse degrees of impact on other industries. Therefore, prioritizing those technical fields that give greater impact in the allocation of subsidies is considered efficient. Meanwhile, the sluggish TFP might have been caused in part by inadequate allocation of even efficient subsidies. In the future, therefore, understanding the relative extent of technical knowledge spillover in each industry before implementing any policy of appropriate R&D subsidies is necessary.

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9Part of the technical knowledge is protected for a certain period as intellectual property such as patents and utility models and is usually not available for use unless payment for a license is made. However, we can see cases where technical knowledge spreads easily in such a way that does not require payment. Suppose, for example, that a firm has developed a next-generation technology and has succeeded in developing a product from this technology. When this new product becomes available in the market, rival firms are able to learn the technology used for the new product through reverse engineering. In this way, because the non-excludability of technical knowledge is limited, the use of the knowledge without paying a fee to the innovator, that is the spillover of technical knowledge, occurs.
5 Concluding Remarks

In this study, we have developed a new quantitative approach based on a framework of Bayesian statistical models for regional production functions. As well, by estimating the prefectural production functions using annual data from the Japanese Prefectural Database covering the period 1955-1995, we have in the main examined trends in TFP and the structural features in the Japanese prefectures. Our approach does not estimate the growth rate of TFP as a residual, as might be done in standard growth accounting analysis, nor does it assume strong constrains on the growth rate of TFP, as might be done in the traditional econometric analysis of production functions. The approach that we propose has the following specific characteristics: (i) smoothness priors are used and trends in TFP are estimated based on a Bayesian method; (ii) the effects of a structural change caused by the first oil crisis are examined; and (iii) the values of the elasticity of output with respect to production factors vary among prefectures.

The main results can be summarized as follows: We considered the influences in the first oil crisis and then analyzed structural changes between two sub-periods: Period I (1955-1973) and Period II (1974-1995). Estimating the elasticity of output with respect to production factors, clear differences in the factor elasticity were identified among the regions. Thus our estimation result questions some earlier reports’ assumptions that factor elasticity values among regions are identical. Regarding the variance of factor elasticity among prefectures, the variance of private capital elasticity decreased and the variance of human capital elasticity and public capital elasticity increased during both periods. As well, a high negative correlation was found between private capital elasticity and public capital elasticity in both periods, which suggested a regional characteristic of Japan that, in general, the lower the private capital elasticity in the region, the higher the public capital elasticity.

Further, we applied the smoothness priors approach to examine the TFP trend in each prefecture during the period 1955-1995. As a result, we found that the TFP in each prefecture increased from the mid-1950s to the early 1970s. During the period between the mid-1970s and the mid-1990s, however, a slowdown in the TFP growth was identified. In particular, TFP trends declined after the 1990s. We can therefore interpret that, in Japan, although TFP promoted economic growth from the mid-1950s to the early 1970, its contribution diminished during the period from mid-1970s to the mid-1990s. A slow decline in the TFP trends was apparent.
particularly after the 1990s, and the effects of inefficiency in resource allocation in
the production factor and financial markets associated with the bad-loan problems
during this period might have been background factors for this situation.

Finally, we refer to some future issues. Although we used the Japanese Prefec-
tural Database, we must bear in mind that these data do not include the capital
utilization ratio or working hours. Therefore, we will attempt to reconsider the re-
sults here in future work after taking into account the capital utilization ratio and
working hours and by utilizing data extended to the most recent possible year. In
addition, an important issue is the specification of production function, and we need
to examine more general cases such as the CES production function.

References


Groot, D. V. Lindley, and A. F. M. Smith, eds., *Bayesian Statistics*, University


Japan-United States Comparison,” *Journal of the Japanese and International


2005.


