Accounting for Economic Growth in Japan and the United States: A Bayesian Approach

Hideo Noda*

Koki Kyo†

Research Group of Economics and Management
No. 2009-E03
2009.6

* Yamagata University (E-mail: noda@human.kj.yamagata-u.ac.jp)
† Obihiro University of Agriculture and Veterinary Medicine (E-mail: kyo@obihiro.ac.jp)

Accounting for Economic Growth in Japan and the United States: A Bayesian Approach*

Hideo Noda†
Faculty of Literature and Social Sciences, Yamagata University, Japan

Koki Kyo
Obihiro University of Agriculture and Veterinary Medicine, Japan

Abstract

We present a Bayesian model for statistical inference in the CES production function, and its application to the empirical analysis of economic growth in Japan and the United States. Unlike the associated literature, we regard the efficiency parameter to be time-varying. By estimation of the time-varying efficiency parameter using a Bayesian method, thus, we attempt to grasp the changes in efficiency at macroeconomic level. It is found that Japan's efficiency performance has stagnated during the 1990s in particular, while in the United States it has risen. The empirical evidence suggests that a reform of national innovation systems is a key issue in restoring Japan's economic growth.

Keywords: CES production function, Box-Cox Transformation, Smoothness Priors, Bayesian Model Averaging, Economic Growth

JEL classification: C11, C22, C51, O40, O57

---

*An earlier version of this paper was presented at the 66th Annual Meeting of the Japan Economic Policy Association, May 2009. We are grateful to Terukazu Suruga for his useful comments. This study was supported by Grant-in-Aid for Scientific Research (C) (21530193) from the Japan Society for the Promotion of Science and by Cooperative Research Program (2016) from the Institute of Statistical Mathematics.

†Corresponding author. E-mail: noda@human.kj.yamagata-u.ac.jp

1 Introduction

Most people, not only economists and policymakers, are interested in economic growth in countries, because trends in economic growth have a direct impact on people's standard of living. In particular, the recent decline in the economic performance of many developed countries such as Japan and the United States necessitates policy recommendations for the recovery of economic growth. In an attempt to respond such social demands, evidence from empirical studies becomes essential, and statistical methods for quantitative analyses of economic fluctuations take on an increasingly important role in policy science. In the present paper, we propose an extended production function approach for the quantitative analysis of economic growth from a Bayesian perspective. Furthermore, we apply our proposed approach to an empirical analysis of the Japanese and U.S. economies.

Overall, earlier empirical studies on sources of growth in a macro economy can be classified into two types. The first is a growth accounting approach, which provides a decomposition of the growth rate of output into components associated with changes in factor inputs, and in a residual that reflects technological progress and other elements (e.g., Young, 1995; Collins and Bosworth, 1996; Wang and Yao, 2003). The second is often called a growth regression approach; it estimates regression equations based on economic growth models (e.g., Barro, 1991; Mankiw, Romer, and Weil, 1992; Shioji, 2001). Nonetheless, these conventional approaches are considered inadequate, as explained below.

As is well known, growth accounting analyses require data on the elasticity of output with respect to conventional factors of production (like physical and human capital), in addition to data on the output and factors of production. Given the difficulty of measuring the elasticity of output with respect to the factors of production directly, the elasticity is measured indirectly as each relevant factor's share of national income. In other words, growth accounting assumes that factor prices are equal to social marginal products. However, the assumption that factor prices coincide with social marginal products is unlikely to hold at all times in reality. If the deviation between the factor prices and social marginal products is too large to neglect, then the results of growth accounting would naturally lack credibility.

Such assumptions are also made in many growth regression models. In addition, a problem with growth regression approaches is that they usually do not represent the time-variations in variables reflecting technological change, such as total factor productivity (TFP), appropriately. For example, Mankiw, Romer, and Weil (1992) assume that the growth rate of TFP is constant at any point in time. This simplification, which evidently lack flexibility, implies that the behavior of TFP is restricted to exponential growth. Furthermore, it should be noted that many growth regression models assume that an economy is in a steady-state. However, as stated in Galor and Zang (1997), it is hard to verify whether an economy is in a steady-state or in a transition path.

As pointed out by Barro (1999), the advantage of the direct estimation of production function models is that it requires none of the assumptions mentioned above. Nevertheless, the conventional econometric analyses of production functions are also inadequate. Specifically, as in growth regression approaches, many earlier reports assume that the growth rate of variables reflecting technological change is constant over time. Therefore, there is likely to be room for improvement in this framework in the context of the production function approach.

Considering these issues that have arisen from previous research, we present a Bayesian model for statistical inference in a constant elasticity of substitution (CES) production function, and its application to empirical analysis of economic growth in Japan and the United States. Unlike the associated literature, in our framework the efficiency parameter is regarded as a time-varying parameter from a Bayesian perspective. By estimation of the time-varying efficiency parameter using a Bayesian method, we attempt to grasp the changes in efficiency at macroeconomic level. This application of a Bayesian method allows a rigorous analysis of trends in a factor reflecting technological change, compared with conventional approaches.

The rest of the paper is organized as follows. In Section 2, we construct an analytical framework. Section 3 presents the basic scheme for parameter estimation. In Section 4, we apply our proposed method to the empirical analysis of economic growth in Japan and the United States. Section 5 concludes the paper.

2 Basic Setup

2.1 CES Production Function

In a given discrete time period t, the maximum amount of output Q_t that can be produced depends on both physical capital K_t and human capital H_t according to an aggregate CES production function

$$Q_t = \zeta[\ \delta K_t^{-\rho} + (1-\delta) H_t^{-\rho}\ ]^{-\frac{\nu}{\rho}}, \tag{1}$$

where parameters ζ, δ, ρ and ν represent the efficiency parameter, the distribution parameter, the substitution parameter and the returns to scale parameter respectively. In Eq. (1), we assume that $\widetilde{\gamma} > 0$, $0 \leq \delta \leq 1$, $\rho > -1$, $\nu > 0$.

A major concern for many people researching economic growth is ascertaining changes in the efficiency parameter. Unlike the existing literature, we therefore consider ζ to be a time-varying parameter ζ_t. To construct a statistical model for the CES production function, we rewrite Eq. (1) as

$$Y_t = \frac{\nu}{\rho} - \frac{\nu}{\rho}[\ \delta K_t^{-\rho} + (1-\delta)H_t^{-\rho}\ ]\gamma_t + \varepsilon_t, \tag{2}$$

where $Y_t = (Q_t^{-\rho/\nu} - 1)/(-\rho/\nu)$, $\gamma_t = \zeta_t^{-\rho/\nu}$, and ε_t denotes an error term. That is, Y_t is defined by a Box-Cox transformation of Q_t. The Box-Cox transformation is used here to equalize the variance of ε_t over time, hence we can assume that $\varepsilon_t \sim \mathrm{N}(0, \sigma^2)$ with σ^2 being an unknown parameter. Zellner (1971) made use of the Box-Cox transformation in analyzing production functions, while Chetty and Sankar (1969) introduced a Bayesian method for parameter estimation of the CES production function based on a similar transformation. However, neither paper allowed the efficiency parameter to vary over time.

2.2 Bayesian Modeling via Smoothness Priors Approach

In this paper, we regard the γ_t to be a random variable from a Bayesian perspective, and apply the smoothness priors approach, introduced by Kitagawa and Gersch (1996), in setting up a prior distribution for it. Specifically, a stochastic difference equation of order 2 is used to define a prior distribution for γ_t as follows:

$$\gamma_t - 2\gamma_{t-1} + \gamma_{t-2} = \psi_t, \tag{3}$$

where ψ_t is a Gaussian white noise sequence with $\psi_t \sim \mathrm{N}(0, \sigma^2/d^2)$, where $d > 0$ is an unknown parameter. It is also assumed that ε_t and ψ_t are independent of each other. In order to obtain a proper prior density of γ_t for $t = 1, 2, \ldots, n$, it is required to introduce γ_0 and γ_{-1} as unknown parameters. Thus, we have three new parameters γ_0, γ_{-1} and d.

We consider the parameters δ, ρ, ν, σ^2, γ_0, γ_{-1} and d unknown constants. When the values of these parameters are given, a system of Bayesian linear equations for γ_t's can be constructed based on Eqs. (2) and (3). Then, we can obtain posterior distribution for γ_t's by using the Bayesian linear modeling method proposed by Akaike (1980). To obtain robust estimates, we combine the Bayesian model averaging method with the Akaike's approach.

3 Estimation Procedure

3.1 Basic Scheme

Our parameter estimation scheme is based on the procedure introduced in Jiang (1995), which is an application of Akaike's approach (Akaike, 1980). For parameter estimation, we need to rewrite Eq. (2) as follows:

$$Y_t = \frac{\nu}{\rho} a - \frac{\nu}{\rho} [\ \delta K_t^{-\rho} + (1-\delta) H_t^{-\rho}\ ] \gamma_t + \varepsilon_t, \tag{4}$$

where a is a formal parameter. Eqs. (2) and (4) are equivalent under the constraint

$$a = 1. \tag{5}$$

We temporarily assume that a is a random variable with the prior distribution

$$a = 1 + \xi, \tag{6}$$

where $\xi \sim \mathrm{N}(0, \sigma^2/\lambda^2)$. To keep Eq. (5) it is required to put $\xi = 0$ into Eq. (6). This can be realized by setting the variance $\sigma^2/\lambda^2 = 0$, or equivalently $\lambda^2 = \infty$ for a given value of σ^2, since it is assumed that the mean of ξ is zero. However, the prior distribution for a is then not defined well. This difficulty can be relieved by putting $\lambda = \lambda^*$ with λ^* being a sufficiently large positive number.

From Eq. (4), the likelihood of $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_n)^{\mathtt{t}}$ and the other related parameters is given by

$$f(\boldsymbol{q}|\boldsymbol{\gamma}, \delta, \rho, \nu, a, \sigma^2) = J \left(\frac{1}{\sqrt{2\pi\sigma^2}} \right)^n \exp \left[-\frac{1}{2\sigma^2} ||\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\gamma} - \frac{\nu}{\rho} \mathbf{1}_n a||^2 \right],$$

where J is the Jacobian of the transformation from Q_t to Y_t defined by $J = \Pi_{t=1}^n (\partial Y_t / \partial Q_t)$. The other symbols are defined as follows: $\boldsymbol{q} = (Q_1, Q_2, \ldots, Q_n)^{\mathtt{t}}$, $\boldsymbol{q} = (Y_1, Y_2, \ldots, Y_n)^{\mathtt{t}}$, $\boldsymbol{X} = \mathrm{diag}(x_1, x_2, \cdots, x_n)$ with $x_i = [\delta K_i^{-\rho} + (1-\delta) H_i^{-\rho}] \cdot (\nu/\rho)$, $\mathbf{1}_n$ is a n-dimensional vector with all elements being one, and $|| * ||$ denotes the Euclidian norm. Furthermore, when the values of σ^2, γ_0, γ_{-1}, d and λ are given, from Eqs. (3) and (6) the prior densities for $\boldsymbol{\gamma}$ and a are given by

$$\begin{aligned} \pi_1(\boldsymbol{\gamma}|\sigma^2, \boldsymbol{b}, d) &= \left(\frac{d}{\sqrt{2\pi\sigma^2}} \right)^n \exp \left[-\frac{d^2}{2\sigma^2} ||\boldsymbol{D}\boldsymbol{\gamma} + \boldsymbol{B}\boldsymbol{b}||^2 \right], \\ \pi_2(a|\sigma^2) &= \left(\frac{\lambda}{\sqrt{2\pi\sigma^2}} \right) \exp \left[-\frac{\lambda^{*2}}{2\sigma^2} (a-1)^2 \right], \end{aligned}$$

where

$$\boldsymbol{D} = \begin{bmatrix} 1 & 0 & \cdots & \cdots & \cdots & 0 \\ -2 & 1 & \ddots & & & \vdots \\ 1 & -2 & 1 & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & -2 & 1 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}$$

and $\boldsymbol{b} = (\gamma_{-1}, \gamma_0)^{\mathrm{t}}$. We then put $\boldsymbol{\theta}^{\mathrm{t}} = (\boldsymbol{\gamma}^{\mathrm{t}}, a)$ and obtain the marginal likelihood of the related parameters as

$$\begin{aligned} & L(\sigma^2, \delta, \rho, \nu, \boldsymbol{b}, d) \\ & \quad = \iint \cdots \int f(\boldsymbol{q}|\boldsymbol{\gamma}, \delta, \rho, \nu, a, \sigma^2)\pi_1(\boldsymbol{\gamma}|\sigma^2, \boldsymbol{b}, d)\pi_2(a|\sigma^2) d\boldsymbol{\gamma} da \\ & \quad = J\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n d^n \lambda^* \det(\boldsymbol{W}^{\mathrm{t}}\boldsymbol{W})^{-\frac{1}{2}} \exp\left[-\frac{1}{2\sigma^2}||\boldsymbol{h} - \boldsymbol{W}\widehat{\boldsymbol{\theta}}||^2\right], \end{aligned} \tag{7}$$

where

$$\boldsymbol{W} = \begin{bmatrix} -\boldsymbol{X} & \frac{\nu}{\rho}\boldsymbol{1}_n \\ -d\boldsymbol{D} & \boldsymbol{0}_n \\ \boldsymbol{0}_n^{\mathrm{t}} & \lambda^* \end{bmatrix}, \quad \boldsymbol{h} = \begin{bmatrix} \boldsymbol{y} \\ d\boldsymbol{B}\boldsymbol{b} \\ \lambda^* \end{bmatrix},$$

$\boldsymbol{0}_n$ is the n-dimensional zero vector, and $\widehat{\boldsymbol{\theta}} = (\widehat{\gamma}^{\mathrm{t}}, \widehat{a})^{\mathrm{t}} = (\boldsymbol{W}^{\mathrm{t}}\boldsymbol{W})^{-1}\boldsymbol{W}^{\mathrm{t}}\boldsymbol{h}$ denotes the mean of a posterior density for $\boldsymbol{\theta}$

$$\begin{aligned} & f(\boldsymbol{\theta}|\boldsymbol{q}; \sigma^2, \delta, \rho, \nu, \boldsymbol{b}, d, \lambda) \\ & \quad = \frac{f(\boldsymbol{q}|\boldsymbol{\gamma}, \delta, \rho, \nu, a, \sigma^2)\pi_1(\boldsymbol{\gamma}|\sigma^2, \boldsymbol{b}, d)\pi_2(a|\sigma^2, \lambda)}{\iint \cdots \int f(\boldsymbol{q}|\boldsymbol{\gamma}, \delta, \rho, \nu, a, \sigma^2)\pi_1(\boldsymbol{\gamma}|\sigma^2, \boldsymbol{b}, d)\pi_2(a|\sigma^2, \lambda) d\boldsymbol{\gamma} da}, \end{aligned} \tag{8}$$

given the related parameters. Therefore, the estimates, $\widehat{\sigma}^2$, $\widehat{\delta}$, $\widehat{\rho}$, $\widehat{\nu}$, $\widehat{\boldsymbol{b}}$ and $\widehat{d}$ for the related parameters can be obtained by maximizing the likelihood $L(\sigma^2, \delta, \rho, \nu, \boldsymbol{b}, d)$ in Eq. (7).

Computationally, if the values of δ, ρ, ν and d are given, $\boldsymbol{\gamma}$ together with $\boldsymbol{b}$ and a can be estimated simultaneously by using the least squares method (see for example, Jiang, 1995). That is, when we set

$$\boldsymbol{z} = \begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{0}_n \\ \lambda^* \end{bmatrix}, \quad \boldsymbol{V} = \begin{bmatrix} -\boldsymbol{X} & \boldsymbol{O} & \frac{\nu}{\rho}\boldsymbol{1}_n \\ d\boldsymbol{D} & d\boldsymbol{B} & \boldsymbol{0}_n \\ \boldsymbol{0}_n^{\mathrm{t}} & \boldsymbol{O} & \lambda^* \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \gamma \\ \boldsymbol{b} \\ a \end{bmatrix}$$

with $\boldsymbol{O}$ denoting a zero matrix, the estimate,

$$\begin{aligned} \widehat{\boldsymbol{\beta}}(\delta, \rho, \nu, d) &= (\widehat{\gamma}^{\mathrm{t}}(\delta, \rho, \nu, d), \widehat{\boldsymbol{b}}^{\mathrm{t}}(\delta, \rho, \nu, d), \widehat{a}(\delta, \rho, \nu, d))^{\mathrm{t}} \\ &= (\boldsymbol{V}^{\mathrm{t}}\boldsymbol{V})^{-1}\boldsymbol{V}^{\mathrm{t}}\boldsymbol{z}, \end{aligned}$$

for $\boldsymbol{\beta}$ is obtained, and then the estimate of σ^2 is given by $\widehat{\sigma}^2(\delta, \rho, \nu, d) = (1/n) \cdot ||\boldsymbol{z} - \boldsymbol{V}\widehat{\boldsymbol{\beta}}(\delta, \rho, \nu, d)||^2$. Note that the corresponding estimates $\widehat{\boldsymbol{\gamma}}(\delta, \rho, \nu, d)$, $\widehat{\boldsymbol{b}}(\delta, \rho, \nu, d)$ and $\widehat{a}(\delta, \rho, \nu, d) \approx 1$ of $\boldsymbol{\gamma}$, $\boldsymbol{b}$ and a can be obtained from each corresponding part of $\widehat{\boldsymbol{\beta}}(\delta, \rho, \nu, d)$. Thus, by substituting $\boldsymbol{b} = \widehat{\boldsymbol{b}}(\delta, \rho, \nu, d)$, and $\sigma^2 = \widehat{\sigma}^2(\delta, \rho, \nu, d)$ into $L(\sigma^2, \delta, \rho, \nu, \boldsymbol{b}, d)$, the partial likelihood of δ, ρ, ν and d can be obtained as

$$L^*(\delta, \rho, \nu, d) = \left[\frac{1}{\sqrt{2\pi\widehat{\sigma}^2(\delta, \rho, \nu, d)}}\right]^n d^n \lambda^* \det(\boldsymbol{W}^{\mathrm{t}}\boldsymbol{W})^{-\frac{1}{2}} \exp\Big(-\frac{n}{2}\Big). \tag{9}$$

Correspondingly, a conditional poterior density, $g(\boldsymbol{\gamma}|\boldsymbol{q}; \delta, \rho, \nu, d)$, for $\boldsymbol{\gamma}$ can be derived form $f(\boldsymbol{\theta}|\boldsymbol{q}; \sigma^2, \delta, \rho, \nu, \boldsymbol{b}, d)$ in Eq. (8) by

$$g(\boldsymbol{\gamma}|\boldsymbol{q}; \delta, \rho, \nu, d) = f(\boldsymbol{\theta}|\boldsymbol{q}; \widehat{\sigma}^2(\delta, \rho, \nu, d), \delta, \rho, \nu, \widehat{\boldsymbol{b}}(\delta, \rho, \nu, d), d)$$

under the condition that $a = \widehat{a}(\delta, \rho, \nu, d)$.

3.2 Using Bayesian Model Averaging Approach

Theoretically, the parameters δ, ρ, ν and d can be estimated using a numerical method, such as a quasi-Newton method, by maximizing the partial likelihood $L^*(\delta, \rho, \nu, d)$ in Eq. (9). Depending on the function $L^*(\delta, \rho, \nu, d)$, it may sometimes be hard to use such a numerical method directly to estimate each of the parameters δ, ρ, ν and d.

To avoid this difficulty, we use the Bayesian model averaging approach as follows. Firstly, for a given integer I we take $\delta_i = i/(I+1)$ as a value of δ, so for $i = 1, 2, \ldots, I$ we have I discrete values for δ that are uniformly distributed in the interval $(0, 1)$. On the other hand, by using the basic estimation scheme introduced in the preceding subsection, for a value δ_i of δ together with the given values of ρ, ν and d, we have a set of Bayesian linear models which is denoted by M_i. Thus, we can obtain I sets of Bayesian linear models under $i = 1, 2, \ldots, I$.

From the Bayesian model averaging perspective (see for example, Claeskens and Hjort, 2008), we assume that all of the models $\{\mathrm{M}_1, \mathrm{M}_2, \ldots, \mathrm{M}_I\}$ are uncertain and all of them have equal uncertainties. Thus, we use a uniform prior for the models $\{\mathrm{M}_1, \mathrm{M}_2, \ldots, \mathrm{M}_I\}$ as follows:

$$q(\mathrm{M}_i) = \frac{1}{I}, \quad (i = 1, 2, \ldots, I)$$

where $q(\mathrm{M}_i)$ is the prior probability for model M_i. As shown in the preceding subsection a partial likelihood for model M_i is $L^*(\delta_i, \rho, \nu, d)$, as given in Eq. (9).

Thus, a conditional posterior probability for model M_i is given by

$$\begin{aligned} p(\mathrm{M}_i|\boldsymbol{q};\rho,\nu,d) &= \frac{L^*(\delta_i,\rho,\nu,d)q(\mathrm{M}_i)}{\sum_{l=1}^{I} L^*(\delta_l,\rho,\nu,d)q(\mathrm{M}_l)} \\ &= \frac{L^*(\delta_i,\rho,\nu,d)}{\sum_{l=1}^{I} L^*(\delta_l,\rho,\nu,d)}. \end{aligned}$$

Moreover, from the basic estimation scheme mentioned above, we can see that for given δ_i, ρ, ν and d, the posterior mean of $\boldsymbol{\gamma}$ is $\widehat{\boldsymbol{\gamma}}(\delta_i,\rho,\nu,d)$ is as the posterior mean of $\boldsymbol{\gamma}$, defined by

$$\widehat{\boldsymbol{\gamma}}(\delta_i,\rho,\nu,d) = \int\int\cdots\int \boldsymbol{\gamma} g(\boldsymbol{\gamma}|\boldsymbol{q};\delta_i,\rho,\nu,d)d\boldsymbol{\gamma}.$$

Therefore, based on the Bayesian model averaging approach a synthetic estimate for $\boldsymbol{\gamma}$ is (see, Draper, 1995):

$$\widehat{\widehat{\boldsymbol{\gamma}}}(\rho,\nu,d) = \sum_{l=1}^{I} \widehat{\boldsymbol{\gamma}}(\delta_l,\rho,\nu,d)p(\mathrm{M}_l|\boldsymbol{q};\rho,\nu,d).$$

Now, we consider the quantity

$$\begin{aligned} \bar{L}(\rho,\nu,d) &= \sum_{l=1}^{I} L^*(\delta_l,\rho,\nu,d)q(\mathrm{M}_l) \\ &= \frac{1}{I}\sum_{l=1}^{I} L^*(\delta_l,\rho,\nu,d) \end{aligned} \tag{10}$$

to be a measure of goodness of fit for the model with respect to the parameters ρ, ν and d. Thus, the estimates $\widehat{\rho}$, $\widehat{\nu}$ and $\widehat{d}$ of ρ, ν and d can be obtained by maximizing the mean likelihood $\bar{L}(\rho,\nu,d)$ in Eq. (10) with respect to these parameters. Hence, the final estimate for $\boldsymbol{\gamma}$ is obtained as

$$\widehat{\widehat{\boldsymbol{\gamma}}} = (\widehat{\widehat{\gamma}}_1, \widehat{\widehat{\gamma}}_2, \ldots, \widehat{\widehat{\gamma}}_n)^{\mathtt{t}} = \widehat{\widehat{\boldsymbol{\gamma}}}(\widehat{\rho},\widehat{\nu},\widehat{d}),$$

and the estimate for δ is given by

$$\widehat{\delta} = \sum_{l=1}^{I} \delta_l p(\mathrm{M}_l|\boldsymbol{q};\widehat{\rho},\widehat{\nu},\widehat{d}).$$

As well, from $\gamma_t = \zeta_t^{-\rho/\nu}$, the estimate for the time-varying efficiency parameter ζ_t is given by $\widehat{\zeta}_t = \widehat{\widehat{\gamma}}_t^{-\nu/\rho}$.

4 Empirical Analysis

4.1 Data

Our samples for Japan and the United States cover a 42-year period (1960-2001). We use real GDP as a measure of output. In our statistical analysis, the real GDP (market prices, billions of national currency at 1995 prices), physical capital (billions of national currency, beginning-of-year stock) and the number of employed persons are data presented in Kamps (2006). For the data of physical capital, we use the aggregate of Kamps' (2006) private and public capital to represent physical capital.

The data of human capital resulting from schooling are constructed as follows. As in Hall and Jones (1999) and Caselli (2005), we assume that human capital per worker, $\widetilde{h}$, is measured by the equation

$$\widetilde{h} = \exp[\phi(s)],$$

where s is the average number of years of schooling. As stated in Hall and Jones (1999), in this specification $\phi(s)$ denotes the efficiency of a unit of labor with s years of schooling relative to one with no schooling ($\phi(0) = 0$). The derivative $\phi'(s)$ is the return to schooling estimated in a Mincerian wage regression: an additional year of schooling raises a worker's efficiency proportionally by $\phi'(s)$. For the first four years of schooling, we assume that the rate of return of schooling is 11.7 percent, which corresponds to the average of sub-Saharan African countries reported by Psacharopoulos and Patrinos (2004). We also assume a rate of return of schooling of 9.7 percent for the next four years of schooling, and a rate of return of schooling of 7.5 percent for schooling beyond the eighth year. The values of 9.7 percent and 7.5 percent are, respectively, the average for the world as a whole and the average of OECD countries from Psacharopoulos and Patrinos (2004). Thus, $\phi(s)$ is specified as follows:

$$\phi(s) = \begin{cases} 0.117 \times s & \text{if } s \leq 4, \\ 0.117 \times 4 + 0.097 \times (s-4) & \text{if } 4 < s \leq 8, \\ 0.117 \times 4 + 0.097 \times 4 + 0.075 \times (s-8) & \text{if } s > 8. \end{cases}$$

Therefore, the data for aggregate human capital (skill-adjusted labor) are produced according to $P \cdot \exp[\phi(s)]$, where P denotes the number of employed persons. In the present paper, we use data for s constructed by Klenow and Rodriguez-Clare (2005). Because of source data constraints, however, we could not obtain the data of human capital per worker in 2001 for both Japan and the United States. Consequently, we generate the data for Japan by using a linear extrapolation based on the fact that in both Japan and the United States, human capital per worker increased linearly in the period 1985-2000.

Figure 1 shows the changes in growth rate of GDP, physical capital and human capital from 1961 to 2001 in Japan.

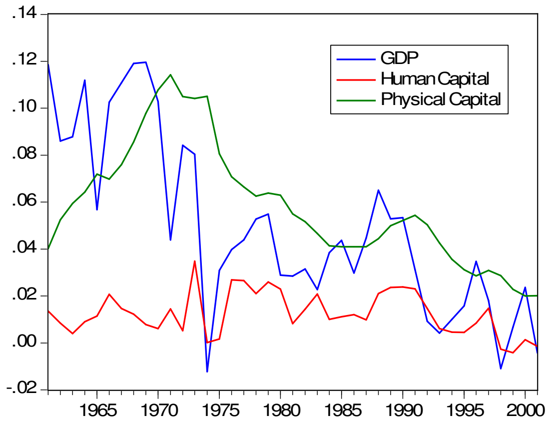

Figure 1: Growth rates of GDP, physical capital and human capital in Japan

In Japan, the average annual growth rates of physical capital during the periods 1960-1969, 1970-1979, 1980-1989 and 1990-2001 are 6.84, 8.57, 4.57 and 3.31 percent, respectively. The average annual growth rates of human capital during these periods are 1.13, 1.73, 1.45 and 0.62 percent, respectively.

Figure 2 displays the changes in growth rates of GDP, physical capital and human capital, respectively, from 1961 to 2001 in United States.

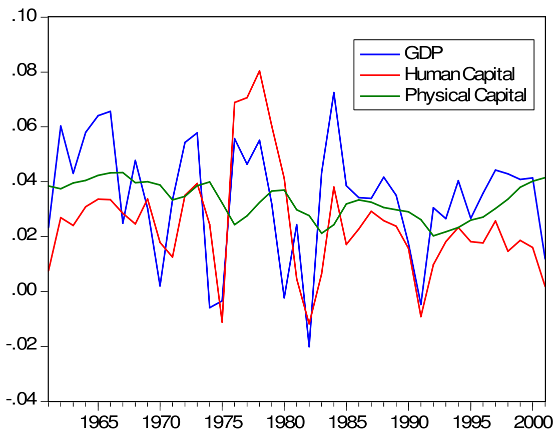

Figure 2: Growth rates of GDP, physical capital and human capital in U.S.

In the United States, the average annual growth rates of physical capital during the periods 1960-1969, 1970-1979, 1980-1989 and 1990-2001 are 4.05, 3.33, 2.90 and 2.98 percent, respectively. The average annual growth rates of human capital during these periods are 2.70, 4.19, 1.73 and 1.41 percent, respectively.

4.2 Estimation Results

Table 1 presents the estimation results of the returns to scale parameter (ν) the distribution parameter (δ) and the substitution parameter (ρ) for Japan and the United States.

Table 1: Estimation results of ν, δ and ρ

	Japan	U.S.
$\widehat{\nu}$	0.950	0.950
$\widehat{\delta}$	0.089	0.248
$\widehat{\rho}$	−0.360	−0.810

For both Japan and the United States, the estimate of the returns to scale parameter is 0.950. Hence, the CES production function in both countries is regarded as decreasing returns to scale during the period 1960-2001. As well, from the estimates of substitution parameters, the elasticities of substitution in Japan and the United Stated are 1.563 and 5.263, respectively. That is, the elasticity of substitution between physical and human capital in the United States is more than three times that of Japan's. Furthermore, Japan's estimate of distribution parameter is very small compared with that of the United States', that is, 0.089 versus 0.248. From Eq. (1), we find that if δ is large, then elasticity of output with respect to physical capital is large and elasticity of output with respect to human capital is small. Therefore, all other things being equal, Japan's small estimate of distribution parameter suggests that elasticity of output with respect to physical capital is small in comparison with that of the United States.

Table 2 shows the estimates of the efficiency parameter (ζ_t) for Japan and the United States.

We set $\zeta_t^* = \log(\widehat{\zeta}_t/\widehat{\zeta}_1)$ to be the efficiency index at time t, so that ζ_1^* corresponds to the value of the efficiency index in 1960. Figures 4 and 5 illustrate the changes in efficiency indices from 1960 to 2001 in Japan and the United States, respectively.

As shown in Figure 4, the efficiency index for Japan rose rapidly during the 1960s. When we consider the factors underlying this trend, the steady increase might be

Table 2: Estimation results of ζ_t

	Japan	U.S.		Japan	U.S.
1960	1.2546	0.0376	1981	3.3892	0.0417
1961	1.3713	0.0385	1982	3.4197	0.0420
1962	1.4844	0.0395	1983	3.4512	0.0431
1963	1.6055	0.0405	1984	3.5167	0.0444
1964	1.7198	0.0416	1985	3.5988	0.0455
1965	1.8285	0.0427	1986	3.6789	0.0461
1966	1.9626	0.0437	1987	3.7850	0.0466
1967	2.1328	0.0443	1988	3.9170	0.0471
1968	2.3369	0.0447	1989	4.0340	0.0476
1969	2.5554	0.0448	1990	4.1194	0.0479
1970	2.7449	0.0449	1991	4.1422	0.0482
1971	2.8803	0.0454	1992	4.1178	0.0488
1972	3.0087	0.0460	1993	4.0916	0.0495
1973	3.0869	0.0464	1994	4.0935	0.0501
1974	3.0917	0.0462	1995	4.1305	0.0508
1975	3.1077	0.0459	1996	4.1871	0.0517
1976	3.1400	0.0455	1997	4.2023	0.0527
1977	3.1906	0.0447	1998	4.1882	0.0540
1978	3.2588	0.0438	1999	4.2057	0.0553
1979	3.3228	0.0427	2000	4.2448	0.0565
1980	3.3585	0.0418	2001	4.2531	0.0573

mainly reflected in the technological changes arising out of the technology imported from North America and Europe. Since the 1970s, however this growth rate has declined sharply, particularly from the 1990s onward. As is well documented, from the mid-1970s, a number of Japanese industries became the owners of leading-edge global technology. This meant that the pace of imitation-based technological improvement declined, and the development of original state-of-art technology in Japan increased. However, independently creating new technology compared with introducing imitation technology requires large research and development budgets, and also has a higher probability of failure, leading to a decreased rate of technological progress. The average annual growth rates of the efficiency index for Japan during 1960-1969, 1970-1979, 1980-1989 and 1990-2001 are 8.23, 2.15, 2.06 and 0.29 percent, respectively. On the other hand, as shown in Figure 5, the efficiency index for the United States rose moderately during the 1960s, then gradually declined during the 1970s. Overall, however, there is an upward tendency from the 1980s onward. For the United States, the average annual growth rates of the efficiency index in the

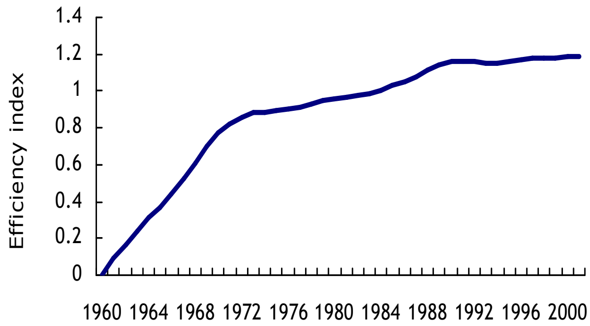

Figure 3: Efficiency index in Japan

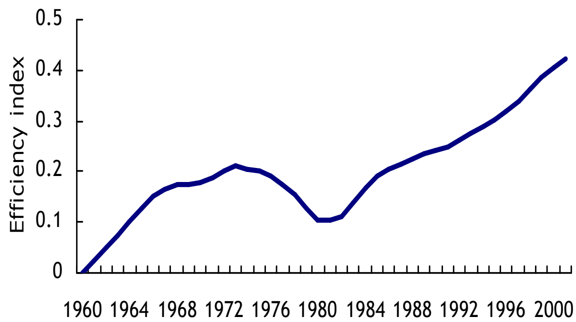

Figure 4: Efficiency index in U.S.

four relevant sub-periods are as follows: 1.97 percent for 1960-1969; -0.56 percent for 1970-1979; 1.46 percent for 1980-1989; 1.65 percent for 1990-2001.

To examine the efficiency index further, we focus attention on the data of Japan's technological knowledge stock obtained from the Cabinet Office (2002). The technological knowledge stock is defined as the accumulated technological knowledge of the enterprises which have been pursuing research and development (R&D). We have the data of Japan's technological knowledge stock for 1973-2000. Let ω_t be technological knowledge stock. We define $\omega_t^* = \log(\omega_t/\omega_1)$ at time t. It follows that $\omega_1^* = 0$, which corresponds to the value in 1973. In addition, we compute $\zeta_t^* = \log(\zeta_t/\zeta_1)$ using the estimates of ζ_t for 1973-2000. Figure 5 depicts the relationship between the efficiency index and the technological knowledge stock.

As shown by the scatter plot in Figure 5, there is a high positive correlation between the efficiency index and the technological knowledge stock; the coefficient of correlation being 0.97. Therefore, we infer that Japan's efficiency index may reflect the effectiveness of investment in R&D and the government's innovation policies.

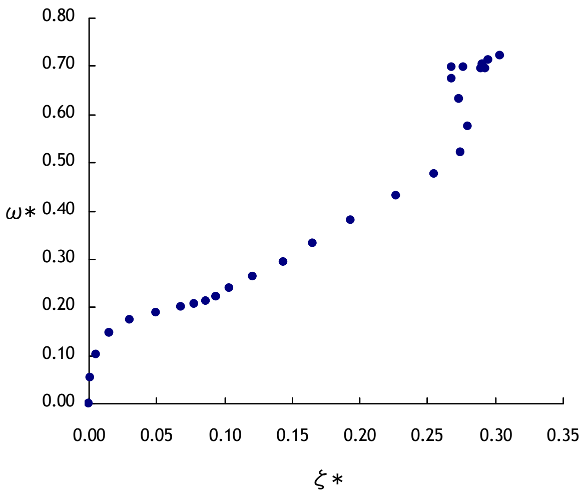

Figure 5: The efficiency index and the technological knowledge stock in Japan

5 Concluding Remarks

In the preceding sections, we have presented an extended production function approach based on Bayesian modeling and its applications to empirical analysis of Japan and the United States. Unlike earlier studies, in our framework the efficiency parameter is regarded as being time-varying rather than constant. By estimating the time-varying efficiency parameter using a Bayesian method, we attempt to understand the changes in efficiency performance at macroeconomic level. Our approach enables a rigorous investigation of the dynamic behavior of a technical factor that contributes to economic growth.

The main results can be summarized as follows. The average annual growth rates of the efficiency index for Japan during 1960-1969, 1970-1979, 1980-1989 and 1990-2001 are 8.23, 2.15, 2.06 and 0.29 percent, respectively. On the other hand, for the United States, the average annual growth rates of the efficiency index in the four relevant decades are: 1.97 percent for 1960-1969; -0.56 percent for 1970-1979; 1.46 percent for 1980-1989; and 1.65 percent for 1990-2001. Taking a look at the average growth rates of the efficiency index in both countries since the 1980s, the efficiency performance in Japan has declined, while in the United States it has risen.

Although an efficiency index includes various components, as mentioned earlier, it may be influenced by the technological factors of industries in an economy. A key constituent of the technological factors is innovation. In connection with the innovative performance and quality of institutions in Japan, for example, Goto (2006) emphasizes not only the research and development of individual companies, but also the importance of reforming the national systems of innovation, such as national institutions, customs, and policies, which together form the basis of research and development. As well, OECD (2006) points out that Japan is lagging behind the U.S. and Europe in the reformation of innovation systems that correspond to current changes. Our empirical evidence supports a policy proposal by OECD (2006) that the reforms of national innovation systems is an important issue for restoring Japan's economic growth.

References

[1] Akaike, H., 1980, Likelihood and the Bayes Procedure, in J. M. Bernardo, M. H. de Groot, D. V. Lindley and A. F. M. Smith, eds., *Bayesian Statistics*, University Press, 143-166.

[2] Barro, R. J., 1991, Economic Growth in a Cross Section of Countries, *Quarterly Journal of Economics*, 106, 407-443.

[3] Barro, R. J., 1999, Notes on Growth Accounting, *Journal of Economic Growth*, 4, 119-137.

[4] Caselli, F., 2005, Accounting for Cross-Country Income Differences, in Aghion, P. and S. Durlauf eds., *Handbook of Economic Growth*, 1A, Elsevier, 679-741.

[5] Chetty, V. K. and U. Sanker, 1969, Bayesian Estimation of the CES Production Function, *Review of Economic Studies*, 36, 289-294.

[6] Claeskens, G. and N. L. Hjort, 2008, *Model Selection and Model Averaging*, Cambridge University Press.

[7] Collins, S. M. and B. P. Bosworth, 1996, Economic Growth in East Asia: Accumulation versus Assimilation, *Brookings Papers on Economic Activity*, 2, 135-203.

[8] Draper, D., 1995, Assessment and Propagation of Model Uncertainty, *Journal of the Royal Statistical Society*, B-57, 45-97.

[9] Galor, O. and H. Zang, 1997, Fertility, Income Distribution, and Economic Growth: Theory and Cross-Country Evidence, *Japan and the World Economy*, 9, 197-229.

[10] Goto, A., 2006, Japan's Innovation System: Its Strengths and Weaknesses, in Goto, A. and T. Kodama eds., *Japan's National Innovation System: Rebuilding the Engine of Growth*, University of Tokyo Press, 1-17. (in Japanese)

[11] Hall, R. E. and Jones, C. I., 1999, Why Do Some Countries Produce So Much More Output Per Worker Than Others?, *Quarterly Journal of Economics*, 114, 83-116.

[12] Jiang, X. Q., 1995, A Bayesian Method for the Dynamic Regression Analysis, *Transactions of the Institute of Systems*, Control and Information Engineers, 8, 8-16.

[13] Kamps, C., 2006, New Estimates of Government Net Capital Stocks for 22 OECD Countries, 1960-2001, *IMF Staff Papers*, 53, 120-150.

[14] Kitagawa, G. and W. Gersch, 1996, *Smoothness Priors Analysis of Times Series*, Springer-Verlag.

[15] Klenow, P. and A. Rodriguez-Clare, 2005, Externalities and Growth, in Aghion, P. and S. Durlauf eds., *Handbook of Economic Growth*, 1A, Elsevier, 817-861.

[16] Mankiw, N. G., D. Romer, and D. N. Weil, 1992, A Contribution to the Empirics of Economic Growth, *Quarterly Journal of Economics*, 107, 407-437.

[17] OECD, 2006, *OECD Economic Surveys Japan 2006*, OECD Publishing.

[18] Psacharopoulos, G. and H. A. Patrinos, 2004, Returns to Investment in Education: A Further Update, *Education Economics*, 12, 111-134.

[19] Shioji, E., 2001, Public Capital and Economic Growth: A Convergence Approach, *Journal of Economic Growth*, 6, 205-227.

[20] Wang, Y. and Y. Yao, 2003, Sources of China's Economic Growth 1952-1999: Incorporating Human Capital Accumulation, *China Economic Review*, 14, 32-52.

[21] Young, A., 1995, The Tyranny of Numbers: Confronting the Statistical Realities of the East Asian Growth Experience, *Quarterly Journal of Economics*, l10, 641-680.

[22] Zellner, A., 1971, *An Introduction to Bayesian Inference in Econometrics*, John Wiley & Sons Inc.