The Inhibitory Effect of Population Ageing on Technical Progress

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Abstract

This article aims to clarify the relationship between population ageing and technical progress. Our discussion is carried out in terms of a model of endogenous growth with expanding product variety. The model implies that the rate of innovation will decline with population ageing in the long term. This phenomenon can be interpreted as ageing having an inhibitory effect on technical progress and thus economic growth. In addition, we find that raising the mandatory retirement age has a positive effect on technical progress. This suggests that encouraging the employment of elderly people is important from the perspective of promoting economic growth.

Keywords: Knowledge-based Economy; Population Ageing; R&D; Technical Progress; Economic Growth

JEL classification: E60; J10; J24; O31; O38

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1 Introduction

Many developed countries are facing the inexorable progress of population ageing. The level of Japan's ageing, in particular, is much higher than that of other developed countries. Moreover, it has been progressing rapidly. Such a demographic phenomenon raises the question: What effects does population ageing have on the trends of the future macro economy? Although various arguments, both optimistic and pessimistic, have been advanced in this regard, no clear consensus has been reached to date. Therefore, the ageing of population is a long-term major concern for economists and policy makers. Incidentally, as exemplified using terms such as knowledge-based economy, the economic activities of major industries in developed countries are becoming increasingly knowledge-intensive today. A salient feature of a knowledge-based economy is that research and development (R&D) activities in response to perceived profit opportunities create innovation, which is accompanied by the accumulation of technological knowledge that has an important role in long-term economic growth.

Considering the social and economic conditions described above, population ageing and innovation are important concepts that cannot be ignored in the analyses of modern economic policies. Nevertheless, theoretical studies on the link between population ageing and innovation are extremely rare. Earlier economic analyses of ageing are dominated by themes related to social security, such as public pensions and health insurance. Social security is unquestionably relevant to ageing. However, the ageing of a population and maturing of a knowledge-based economy progress simultaneously, and gaining insight into the effect of ageing on innovation and thus economic growth is therefore crucial to effective public policy research. The purpose of this article is to clarify the effect of population ageing on the long-term performance of a knowledge-based economy in the context of a growth model with endogenous innovation. That is, this article is concerned with the relationship between population ageing and economic growth, not social security problems.

Recent works from an endogenous growth theory perspective can be classified roughly into two types. The first type focuses on the role of innovation in economic growth. Futagami and Nakajima (2001) used an extended life cycle growth model to show that population ageing is not necessarily a negative factor for economic growth. In their model, the engine of long-term growth is learning-by-doing, which can be interpreted as a kind of incremental improvement innovation. Also, Futagami, Iwaisako and Nakajima (2002) used a model of product innovation through industrial R&D to confirm that population ageing would have a negative effect on macroeconomic growth. The second type stresses human capital. For example, Gruescu (2007, Ch.7) extended the growth model of Lucas (1988) emphasising the workers' skill and its externality. As a result, Gruescu's (2007, Ch.7) model predicts lower economic growth in an economy if the population is ageing.

The marked difference between this article and the existing literature is that it considers the roles of both human capital and innovation. As OECD (2001) and Psarras (2007) argue, in recent years an improvement in workers' skills has been demanded more than ever in developed countries. Therefore, the importance of human capital formation has been increasing further as the economic activities of industries become more knowledge-intensive. Such a tendency suggests the necessity of building a framework that treats human capital in addition to innovation as endogenous variables of the model and reflects the reality of a knowledge-based economy. However, most earlier studies emphasise either innovation or human capital, not both. As a result, the modelling of those earlier studies inadequately reflects the essential characteristics of a knowledge-based economy. We therefore conclude that the results of the analysis presented in the relevant literature described above lack sufficient persuasiveness.

Our model is based on that of Grossman and Helpman (1991, Ch.5), even though they did not examine the relationship between population ageing and technical change. By extending their model we present a manageable framework for analysing macroeconomic effects of population ageing. A main finding derived from our model is that the rate of innovation will decline with population ageing in the long term. This phenomenon can be interpreted as ageing having an inhibitory effect on technical progress and therefore also on economic growth in a knowledge-based economy. In addition, we find that raising the mandatory retirement age has a positive effect on technical progress. Thus, the model implies that encouraging the employment of elderly people is important from the viewpoint of promoting economic growth.

The article is organised as follows. In Section 2 we introduce a model of economic growth through innovation and analyse the relationship between population ageing and technical progress. In Section 3 we examine the policy implications derived from the model. Finally, in Section 4 the main results and conclusions are presented.

2 The Model

2.1 Households

Let us start by describing the setup of the model for households. We consider a representative agent who lives for a finite period of time T. In this economy, individuals in age groups ranging from 0 to T are distributed continuously at each point in time; the density of individuals in each age group is N/T. We also assume that the number of births is the same as the number of deaths at each point in time. Thereby, the total population N is constant. Based on these assumptions, representing age by q, the density function $f(q)$ of age is:

$$f(q) = \begin{cases} \frac{N}{T} & (0 \leq q \leq T), \\ 0 & (\text{otherwise}). \end{cases}$$

Two worker types are considered. One is an individual equipped with skills to engage in R&D activities. The other is an individual without such skills. In the model, individuals of the first type are called skilled workers, and those of the second type are called unskilled workers. For convenience, the set of the worker types is expressed as $\mathcal{B}$, including skilled workers as h, and unskilled workers as l, that is, $\mathcal{B} = \{h, l\}$. We also assume that all individuals retire mandatorily at the age of $Z(< T)$. They work until the mandatory retirement age of Z irrespective of whether they are skilled or unskilled. This can be interpreted as all individuals retiring at the same age Z. We treat N, T, and Z as parameters for the model.

In the model, we assume a closed economy, whose industries comprise two sectors, the nondurable consumer goods sector and the R&D sector. It is assumed that unskilled workers supply labour to the consumer goods sector and that skilled workers supply labour to the R&D sector. We regard unskilled labour as the numéraire, so that the wage rate paid for a unit of unskilled labour is normalised to 1. Skilled workers are paid for embodied human capital at wage rate $w(t)$ at time t.

We analyse the consumer behaviour of households based on the settings above. People who are born at time point κ are called generation κ. The lifetime utility $U_i(\kappa)$ of an individual of type i from generation κ is given by

$$U_i(\kappa) = \int_{\kappa}^{\kappa+T} e^{-\rho(t-\kappa)} \ln D_i(\kappa, t) dt. \tag{1}$$

The parameter ρ denotes the subjective discount rate, and it is assumed that $\rho > 0$. In addition, $\ln D_i(\kappa, t)$ is the instantaneous utility of an individual of type i from generation κ at time t. The type of consumer goods is assumed to be a continuous quantity. The consumer goods that are available at time t are specified as the interval $[0, A(t)]$. Thus, $A(t)$ denotes the measure of consumer goods invented before time t. Households recognise each of the consumer goods as different items, and so product differentiation is assumed. The consumption index, $D_i(\kappa, t)$, is formulated as

$$D_i(\kappa, t) = \Big[\int_0^{A(t)} x_{ij}(\kappa, t)^{\alpha} dj \Big]^{\frac{1}{\alpha}}, \tag{2}$$

where $x_{ij}(\kappa, t)$ is the amount of the j-th product consumed by an individual of type i from generation κ at time t. In addition, the parameter α takes values between 0 and 1. The elasticity of substitution between any two consumer goods is $1/(1-\alpha)$.

Next, we represent the budget constraints that households face. We assume that households may freely lend and borrow at an instantaneous interest rate of $r(t)$, and that no assets or debts remain at the time of death. For an individual of type i from generation κ, total expenditure is expressed as $E_i(\kappa, t)$, and the sum of the discounted present values of income flow acquired is expressed as $W_i(\kappa)$, so the intertemporal budget constraint of an individual of type i from generation κ is given by

$$\int_{\kappa}^{\kappa+T} e^{-\int_{\kappa}^{t} r(s)ds} E_i(\kappa, t) dt \leq W_i(\kappa). \tag{3}$$

The constrained optimisation problem for an individual of type i from generation κ can be considered using the following two steps. The first step is to find the quantity of each type of goods demanded that will maximise the instantaneous utility for given expenditure and prices. In other words, a household of type i from generation κ will solve the following problem:

$$\max \quad \Big[\int_0^{A(t)} x_{ij}(\kappa, t)^{\alpha} dj \Big]^{\frac{1}{\alpha}},$$

$$\text{s.t.} \quad \int_0^{A(t)} p_j(t) x_{ij}(\kappa,t) dj \le E_i(\kappa,t).$$

Solving this problem yields

$$x_{ij}(\kappa,t) = \frac{p_j(t)^{\frac{1}{\alpha-1}} E_i(\kappa,t)}{\displaystyle\int_0^{A(t)} p_m(t)^{\frac{\alpha}{\alpha-1}} dm}. \tag{4}$$

We express the number of households of type i from generation κ in the subjective equilibrium as $M_i(\kappa,t)$. On the assumption that each age group is distributed uniformly at N/T, the relation $\sum_{i\in\mathcal{B}} M_i(\kappa,t) = N/T$ holds for all generations at each point in time. The aggregate demand of households from generation κ for the j-th product is $x_j(\kappa,t)$, and the aggregate expenditure of households from generation κ is $E(\kappa,t)$. That is, $x_j(\kappa,t)$ and $E(\kappa,t)$ are respectively given by

$$x_j(\kappa,t) = \sum_{i\in\mathcal{B}} M_i(\kappa,t) x_{ij}(\kappa,t), \tag{5}$$

$$E(\kappa,t) = \sum_{i\in\mathcal{B}} M_i(\kappa,t) E_i(\kappa,t). \tag{6}$$

Using Eqs. (4)-(6), the aggregate demand of households from generation κ for the j-th product, $x_j(\kappa,t)$, can be derived as

$$x_j(\kappa,t) = \frac{p_j(t)^{\frac{1}{\alpha-1}} E(\kappa,t)}{\displaystyle\int_0^{A(t)} p_m(t)^{\frac{\alpha}{\alpha-1}} dm}.$$

When we express the aggregate demand for the j-th product as $x_j(t)$ and the aggregate expenditure of households in the economy as $E(t)$, we obtain

$$\begin{aligned} x_j(t) &= \int_{t-T}^{t} x_j(\kappa,t) d\kappa \\ &= \frac{p_j(t)^{\frac{1}{\alpha-1}} \displaystyle\int_{t-T}^{t} E(\kappa,t) d\kappa}{\displaystyle\int_0^{A(t)} p_m(t)^{\frac{\alpha}{\alpha-1}} dm} \\ &= \frac{p_j(t)^{\frac{1}{\alpha-1}} E(t)}{\displaystyle\int_0^{A(t)} p_m(t)^{\frac{\alpha}{\alpha-1}} dm}, \end{aligned} \tag{7}$$

where $E(t) \equiv \int_{t-T}^{t} E(\kappa,t) d\kappa$. Note that the price elasticity of the demand for the j-th product is $1/(1-\alpha)$.

The second stage of the problem is to find the path of expenditure that maximises the lifetime utility in Eq. (1), subject to the intertemporal budget constraint in Eq. (3). Substitution from Eq. (4) into Eq. (2) gives

$$D_i(\kappa, t) = \frac{E_i(\kappa, t)}{\left[\int_0^{A(t)} p_j(t)^{\frac{\alpha}{\alpha-1}} dj\right]^{\frac{\alpha-1}{\alpha}}}. \tag{8}$$

To simplify this expression, the denominator in Eq. (8) is denoted by $P_D(t)$, which can be regarded as a kind of price index. That is,

$$P_D(t) \equiv \left[\int_0^{A(t)} p_j(t)^{\frac{\alpha}{\alpha-1}} dj\right]^{\frac{\alpha-1}{\alpha}}.$$

Furthermore, we take the natural logarithms of both sides of Eq. (8):

$$\ln D_i(\kappa, t) = \ln E_i(\kappa, t) - \ln P_D(t).$$

The optimisation problem at the second stage, therefore, is

$$\max \int_\kappa^{\kappa+T} e^{-\rho(t-\kappa)}[\ln E_i(\kappa, t) - \ln P_D(t)]dt,$$
$$\text{s.t.} \int_\kappa^{\kappa+T} e^{-\int_\kappa^t r(s)ds} E_i(\kappa, t)dt \leq W_i(\kappa).$$

As a result, we find that the expenditure growth rate is given by

$$\frac{dE_i(\kappa, t)/dt}{E_i(\kappa, t)} = r(t) - \rho. \tag{9}$$

2.2 Producers of Consumer Goods

In the consumer goods market, there are numerous firms, and each firm leaves the effect of decision making on other firms out of consideration. We assume that all firms in the consumer goods sector are able to produce goods by purchasing patent rights to the exclusive use of the product technology from the R&D firms. As described in Subsection 2.1, various consumer goods supplied to the market have physical differences and consumers recognise such differences. Under this product differentiation, each firm producing a specific consumer good has some control over its own product price. Accordingly, the structure of the consumer goods market is characterised by monopolistic competition.

The input of unskilled labour of the firm that produces the j-th product (called firm j from here on) is L_j. The following relation is assumed to hold between the amount of the j-th product produced and the input of unskilled labour of firm j:

$$x_j = \frac{1}{\zeta} L_j, \tag{10}$$

where $\zeta(> \alpha)$ is a positive constant. Evidently, the smaller the value of ζ, the higher the productivity of unskilled labour of firm j. Because the wage rate of unskilled labour is 1, the cost of input of unskilled labour is $1 \times L_j = L_j$. Using Eq. (10), the profit π_j of firm j is given by

$$\begin{aligned} \pi_j(t) &= p_j(t)x_j(t) - L_j(t) \\ &= [p_j(t) - \zeta]x_j(t). \end{aligned} \tag{11}$$

The demand function for the product produced by firm j is given by Eq. (7). Substituting Eq. (7) into Eq. (11) yields

$$\pi_j(t) = [p_j(t) - \zeta] \frac{p_j(t)^{\frac{1}{\alpha-1}} E(t)}{\int_0^{A(t)} p_m(t)^{\frac{\alpha}{\alpha-1}} dm}. \tag{12}$$

The price that maximises the profit of firm j is given by

$$p_j(t) = \frac{\zeta}{\alpha} \equiv p. \tag{13}$$

Apparently, the price ζ/α exceeds the marginal cost ζ. Equation (13) implies that the markup ratio is $(1-\alpha)/\alpha$. If we substitute Eq. (13) into Eq. (7), then the quantity of the j-th product demanded is

$$x_j(t) = \frac{\alpha E(t)}{\zeta A(t)} \equiv x(t). \tag{14}$$

Furthermore, substitution of Eqs. (13) and (14) into Eq. (11) leads to

$$\pi_j(t) = \left(\frac{\zeta - \alpha}{\zeta}\right) \frac{E(t)}{A(t)} \equiv \pi(t). \tag{15}$$

Let $V_j(t)$ be the discounted present value of the profit flow gained by firm j at time t. That is, $V_j(t)$ is given by

$$V_j(t) = \int_t^{\infty} e^{-\int_t^{\tau} r(\iota)d\iota} \pi(\tau) d\tau \equiv V(t).$$

2.3 R&D Firms

Suppose that R&D firms are continuously distributed in an interval $[0, 1]$. We consider the consumer goods design developer and consumer goods producer to be separate agents. Without considering the uncertainty in R&D activities, it is assumed that new technology is definitely created through the input of human capital. Thus, we use a deterministic framework for the invention of new products. The setup of the R&D sector in our model is similar to those in Romer (1990) and Grossman and Helpman (1991, Ch.3).

We assume that if an R&D firm $f \in [0, 1]$ inputs H_f units of human capital, then it develops new product designs of Δa_f during the period Δt. More specifically, the relation $\Delta a_f = \lambda \Omega H_f \Delta t$ holds. Here Ω denotes public knowledge capital and λ is a positive constant. Knowledge capital refers to the set of technological information that has been accumulated through R&D activities. It should be noted that the contribution of public knowledge capital (Ω) to the creation of new product designs (Δa_f) reflects the technological spillover by partial non-excludability of knowledge capital.

Existing technological information offers wide applicability and helps product planners and product managers during the development of new product concepts. Such explicit knowledge may diffuse widely across industries and leads to the formation of public knowledge capital over time. The accumulation of knowledge capital further contributes to the creation of new product design. In fact, even if technology is patented, the appropriability of technology through a patent is not complete, and technological spillover is commonly observed (Odagiri, 2001). A schematic representation of this innovation process is presented in Figure 1.

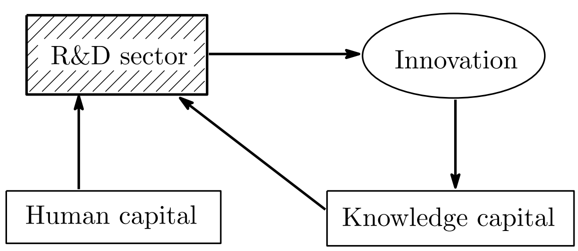

Figure 1: The Innovation Process

The number of new technologies created by R&D firm f at time t is expressed as follows:

$$\dot{a}_f(t) = \lambda\Omega(t)H_f(t), \tag{16}$$

where an overdot represents a time derivative. The sum total of the number of new technologies created by each R&D firm at time t, that is, $\int_0^1 \dot{a}_f(\tau)df$, corresponds to the number of new technologies developed in the macro economy, $\dot{A}(\tau)$. If we assume that $\lim_{t\to-\infty} A(t) = 0$, then $A(t) = \int_{-\infty}^{t} \int_0^1 \dot{a}_f(\tau)df d\tau$ holds.

Grossman and Helpman (1991, Ch.3) considered that the variety of consumer goods reflects the cumulative experience of R&D and regarded it as a proxy variable for knowledge capital. We follow them by assuming $\Omega(t) = A(t)$. Therefore, Eq. (16) can be rewritten as

$$\dot{a}_f(t) = \lambda A(t)H_f(t). \tag{17}$$

An R&D production function as described in Eq. (17) is often called a knowledge-driven specification.

Suppose that an inventor of a new technology is granted a patent immediately by the government and that the duration of the patent is infinite. We simplify the analysis by assuming that the cost of maintaining the patent is zero. Each R&D firm earns revenue by selling its patents to an agent attempting to enter the consumer goods industry.

Recall that, in the subjective equilibrium, the discounted present value of the profit flow for all firms producing consumer goods is equal to $V(t)$. Consequently, the patent price in the general equilibrium of an arbitrary R&D firm is equal to $V(t)$. We prove that the patent price of an arbitrary R&D firm equals $V(t)$ by following the argument of Jones (2002). Assume a situation in which anyone who attempts to enter the consumer goods market can participate in an English auction for patents. Let us imagine that a patent of an R&D firm f is being auctioned. If the starting price is higher than $V(t)$, then there will be no incentive for anyone to bid. The reason is that purchasing the patent and producing the consumer goods would only result in a net loss. If the bid price of a participant is below $V(t)$, then someone will try to bid higher and eventually the bid price will be raised to $V(t)$. Thus, when we consider the equilibrium in which the patent of the R&D firm f is sold and the new patent holder produces the consumer good, the price of the patent of the R&D firm f is expected to be $V(t)$. This logic evidently applies to the transfer price of all other patents. Therefore, in equilibrium, when all types of consumer goods are produced, the transfer price of an arbitrary patent must equal $V(t)$.

The cost of inputting $H_f(t)$ units of human capital is $w(t)H_f(t)$, and the revenue of an R&D firm f is $\dot{a}_k(t)V(t)$. Therefore, if the profit of an R&D firm f, which invents the technology for $\dot{a}_f$ units of the new product at time t, is denoted by $\Pi_f(t)$, then the following holds:

$$\begin{aligned} \Pi_f(t) &= \dot{a}_f(t)V(t) - w(t)H_f(t), \\ &= [\lambda A(t)V(t) - w(t)]H_f(t). \end{aligned} \tag{18}$$

Because each R&D firm constitutes only a small portion of the entire R&D sector, public knowledge capital is considered given for each R&D firm.

Our interest is in the equilibrium in which product innovation occurs in the sense of $\dot{a}_f(t) > 0$. As Eq. (17) shows, the creation of innovation requires the input of human capital. Therefore, the condition $H_f(t) > 0$ must be satisfied in an equilibrium in which innovation occurs. Under the condition $H_f(t) > 0$, Eq. (18) suggests that, when $\lambda A(t)V(t) > w(t)$, the more resources that are used in R&D, the greater the profit will be. In this case, firms have the incentive of trying to input an unlimited amount of human capital in R&D activities. However, input of an infinite amount of human capital is not possible in the equilibrium. When $\lambda A(t)V(t) < w(t)$, Eq. (18) implies that the input of resources into R&D would result in negative profit. In this case, the rational decision for the firm is to not input resources in R&D, that is, $H_f(t) = 0$, but this contradicts $H_f(t) > 0$. Therefore, in the subjective equilibrium for which free entry to the market is guaranteed and product innovation is created, $\lambda A(t)V(t) = w(t)$ must be satisfied. Consequently, the following equation is obtained:

$$A(t)V(t) = \frac{w(t)}{\lambda}. \tag{19}$$

2.4 Market Equilibrium

We follow Grossman and Helpman (1991, Ch.3) by assuming that the stock prices of consumer goods firms are consistent with $V(t)$, which is the discounted present value of the profit flow for all firms producing consumer goods. An individual who owns stocks from t to $t + \Delta t$ receives $\pi(t)\Delta t$ in dividends. Also, capital gains of $\dot{V}(t)\Delta t$ can be earned. Therefore, the revenue that a stockholder can earn during this period is $\pi(t)\Delta t + \dot{V}(t)\Delta t$. Incidentally, when funds of $V(t)$ are lent from t to $t + \Delta t$, the revenue is expressed as $r(t)V(t)\Delta t$. In the equilibrium of the asset market, the revenues of the former and latter must be consistent. Consequently, the following holds:

$$\pi(t) + \dot{V}(t) = r(t)V(t). \tag{20}$$

Equation (20) is called the no-arbitrage condition.

We turn now to an equilibrium condition in the consumer goods market. In the subjective equilibrium of consumer goods firms, the production amount of all firms is $x(t)$. Therefore, the total supply of consumer goods at time t is given by $\int_0^{A(t)} x(t)dj = A(t)x(t)$. On the other hand, the aggregate demand for consumer goods is $\alpha E(t)/\zeta$ from Eq. (14). Consequently, the following equation holds for the equilibrium in the consumer goods market:

$$\frac{\alpha E(t)}{\zeta} = A(t)x(t). \tag{21}$$

Market clearing of unskilled labour requires that

$$\zeta A(t)x(t) = L(t), \tag{22}$$

where $L(t) \equiv \int_0^{A(t)} L_j(t)dj$. The left-hand side of Eq. (22) represents the aggregate demand for unskilled labour and the right-hand side expresses the aggregate supply.

Finally, market clearing of human capital implies that

$$\frac{\dot{A}(t)}{\lambda A(t)} = H(t), \tag{23}$$

where $H(t) \equiv \int_0^1 H_f(t)df$. The left-hand side of Eq. (23) represents the aggregate demand for human capital, while the right-hand side expresses the aggregate supply.

2.5 Steady-State Equilibrium

Let us examine the relationship between the ageing of population and long-term product development performance of the R&D industry. We focus on the steady-state equilibrium of the model and derive the reduced-form equation for the innovation rate. As in Barro and Sala-i-Martin (2004), the steady-state equilibrium is defined as the situation where all economic variables grow at constant rates in the general equilibrium.

The period of an individual's education is $S(t)$. Let $\theta(t)$ be the ratio of the sum of the number of people receiving education, the number of active skilled workers and the number of retired skilled workers to the total population. Recall that the total population at each point in time consists of people receiving education, active unskilled workers, active skilled workers, and retired people. Then the number of people receiving education is $S(t)\theta(t)N/T$; the number of unskilled workers is $[1-\theta(t)]NZ/T$; the number of skilled workers is $(Z-S)\theta(t)N/T$; and the number of retired people is $(T-Z)N/T$. The composition of the total population at each point in time is depicted in Figure 2.

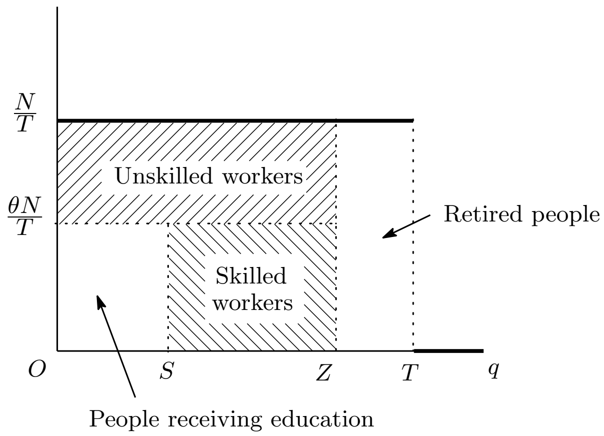

Figure 2: Composition of the Total Population

Incidentally, the equation below holds for the number of unskilled workers:

$$L(t) = [1-\theta(t)]\frac{N}{T}Z. \tag{24}$$

Therefore, the total population N can be expressed as

$$N = S(t)\theta(t)\frac{N}{T} + L(t) + [Z-S(t)]\theta(t)\frac{N}{T} + (T-Z)\frac{N}{T}. \tag{25}$$

Suppose now that the human capital per skilled worker, $h(t)$, depends only on the number of years in education. We specify the function for human capital formation as follows:

$$h = e^{\mu(S)}, \tag{26}$$

where the derivative $\mu'(S)$ represents the gain from the number of additional years in education, that is, the rate of return on investment in education. We follow Lee (2005) by considering the case in which $\mu(S)$ is the linear function $\mu(S) = \mu \times S$. It is assumed that $\mu > 0$ is a constant. Consequently, the following equation holds for the amount of human capital in the macro economy:

$$H(t) = [Z - S(t)]\theta(t)\frac{N}{T}e^{\mu S(t)}. \tag{27}$$

Recall that, by definition, the growth rate of all economic variables is constant in the steady-state equilibrium. First, Eq. (24) implies that $\theta(t)$ and $L(t)$ are constant in the steady-state equilibrium. Also, when $\theta(t)$ and $L(t)$ are constant, $S(t)$ must also be constant from Eq. (25). Furthermore, using Eqs. (21) and (22), we get

$$L(t) = \alpha E(t). \tag{28}$$

It follows that $E(t)$ is also constant. Therefore, when the economy is in a steady-state equilibrium, the aggregate consumption expenditure of the household sector is constant at the level $E > 0$.

We now examine the equilibrium path in which an individual of type i from generation κ maintains a constant consumption level over time. In such a consumption expenditure path, $E_i(\kappa, s) = E_i(\kappa, t)$ holds for all $s \neq t$, and so Eq. (9) implies $r(t) = \rho$. Dividing both sides of Eq. (20) by $V(t)$ gives

$$\frac{\pi(t)}{V(t)} + \frac{\dot{V}(t)}{V(t)} = \rho. \tag{29}$$

Because $\dot{V}(t)/V(t)$ and ρ are constant in Eq. (29), the profit rate $\pi(t)/V(t)$ must also be constant. In addition, Eqs. (19) and (22) imply

$$\frac{\pi(t)}{V(t)} = \Big(\frac{\zeta - \alpha}{\zeta}\Big)\frac{\lambda E}{w(t)}. \tag{30}$$

Equation (30) shows that $w(t)$ is constant. Let $g \equiv \dot{A}(t)/A(t)$ be the rate of innovation in a steady state. Considering Eqs. (19), (28), (29), and (30), we get

$$\Big(\frac{\zeta-\alpha}{\alpha\zeta}\Big)\frac{\lambda L}{w} = g+\rho. \tag{31}$$

Incidentally, an individual has two alternatives: either making an investment in human capital to work as a skilled worker in the subsequent period or working as an unskilled worker without receiving education. If an individual of generation κ engages in labour as an unskilled worker, then the discounted present value of that person's lifetime income, $W_l(\kappa)$, is expressed as

$$\begin{aligned} W_l(\kappa) &= \int_{\kappa}^{\kappa+Z} e^{-\rho(t-\kappa)}dt \\ &= \frac{1-e^{-\rho Z}}{\rho}. \end{aligned} \tag{32}$$

If an individual of generation κ engages in labour as a skilled worker, then the discounted present value of that person's lifetime income, $W_h(\kappa)$, for $S \leq Z$, is given by

$$\begin{aligned} W_h(\kappa) &= \int_{\kappa+S}^{\kappa+Z} e^{-\rho(t-\kappa)} w e^{\mu S} dt \\ &= \frac{(e^{-\rho S}-e^{-\rho Z})we^{\mu S}}{\rho}. \end{aligned} \tag{33}$$

An individual wishing to be a skilled worker faces the problem of selecting the education period with a given time constraint. More specifically, the individual solves the following problem:

$$\begin{aligned} &\max \;\; \chi(S) = \frac{1}{\rho}(e^{-\rho S}-e^{-\rho Z})we^{\mu S}, \\ &\text{s.t.} \;\; S \leq Z. \end{aligned}$$

We assume here that $\lim_{S\to 0} e^{\rho S} > e^{\rho Z}(\mu-\rho)/\mu$. Solving the optimisation problem yields the following interior solution:

$$S = \frac{1}{\rho}\Big[\ln\Big(1-\frac{\rho}{\mu}\Big)+\rho Z\Big]. \tag{34}$$

In the following argument, we define $\nu(\mu,\rho,Z) \equiv (1/\rho)\cdot[\ln(1-\rho/\mu)+\rho Z]$ to simplify the expression.

All households are representative agents who have perfect foresight. They compare the discounted present value of their lifetime income that can be earned if they become skilled workers with that if they become unskilled workers to decide which type of worker they will become. Such selection can be interpreted as the decision making for the sector that supplies labour services, which is whether to engage in labour in the consumer goods sector or in the R&D sector. The subject of our analysis is the steady-state equilibrium in which production activities are conducted in both the consumer goods sector and the R&D sector. If a difference in the discounted present value of lifetime income arises between these sectors, all individuals should attempt to engage in labour in the sector in which they can earn a higher discounted present value of lifetime income. Therefore, in the steady-state equilibrium in which production activities are conducted in both the consumer goods sector and the R&D sector, the discounted present value of lifetime income must be equal between the sectors. This condition means $W_l(\kappa) = W_h(\kappa)$. Using Eqs. (32) and (33), we obtain

$$\frac{1-e^{-\rho Z}}{\rho} = \frac{(e^{-\rho S} - e^{-\rho Z})we^{\mu S}}{\rho}.$$

This equation can be rewritten as follows:

$$w = \frac{1-e^{-\rho Z}}{[e^{-\rho\nu(\mu,\rho,Z)} - e^{-\rho Z}]e^{\mu\nu(\mu,\rho,Z)}}. \tag{35}$$

Therefore, considering Eqs. (24), (31), and (35), we get

$$\rho + g = \Big(\frac{\zeta-\alpha}{\alpha\zeta}\Big)\frac{(1-\theta)\lambda NZ}{T}e^{\mu\nu(\mu,\rho,Z)}\Big[\frac{e^{-\rho\nu(\mu,\rho,Z)} - e^{-\rho Z}}{1-e^{-\rho Z}}\Big]. \tag{36}$$

Combining Eqs. (23) and (27) with (34) yields

$$g = \frac{\lambda[Z-\nu(\beta,\rho,Z)]}{T}\theta N e^{\mu\nu(\mu,\rho,Z)}. \tag{37}$$

Consequently, Eqs. (36) and (37) can be used to calculate the values of g and θ in the steady-state equilibrium. It is found that g in a steady-state equilibrium is given by

$$g = \frac{1}{\phi}\Bigg\{\Big(\frac{\zeta-\alpha}{\alpha\zeta}\Big)\frac{\lambda N}{T}Ze^{\mu\nu(\mu,\rho,Z)}\frac{[e^{-\rho\nu(\mu,\rho,Z)} - e^{-\rho Z}]}{1-e^{-\rho Z}} - \rho\Bigg\}, \tag{38}$$

where ϕ is defined by

$$\phi \equiv 1 + \Big(\frac{\zeta - \alpha}{\alpha\zeta}\Big)\frac{Z[e^{-\rho\nu(\mu,\rho,Z)} - e^{-\rho Z}]}{(1 - e^{-\rho Z})[Z - \nu(\mu,\rho,Z)]}.$$

In the argument below, we call the rate of innovation in a steady-state equilibrium the rate of long-term innovation. Equation (38) implies that, as N decreases, g decreases. Thus, the model predicts that in a knowledge-based economy the rate of long-term innovation will decline with shrinking population.

For θ in a steady-state equilibrium we get

$$\theta = \frac{1}{\psi}\Bigg\{\Big(\frac{\zeta - \alpha}{\alpha\zeta}\Big)Ze^{\mu\nu(\mu,\rho,Z)}\frac{[e^{-\rho\nu(\mu,\rho,Z)} - e^{-\rho Z}]}{1 - e^{-\rho Z}} - \frac{\rho T}{\lambda N}\Bigg\}, \tag{39}$$

where ψ is defined by

$$\psi = [Z - \nu(\beta,\rho,Z)]e^{\mu\nu(\mu,\rho,Z)} + \Big(\frac{\zeta - \alpha}{\alpha\zeta}\Big)\frac{Ze^{\mu\nu(\mu,\rho,Z)}[e^{-\rho\nu(\mu,\rho,Z)} - e^{-\rho Z}]}{1 - e^{-\rho Z}}.$$

3 Policy Implications

3.1 Ageing and Innovation

We first examine the influence of the progress of ageing on innovation and consequent growth performance. In general, population ageing means that older groups in the age structure of a population increase. The percentage (or proportion) of elderly people (usually 65 years old and older) in the total population of a country is often used as a representative indicator of ageing.

In the model, individuals at the age of Z and older are regarded as elderly people, so the total number of individuals at the age of Z and older (the elderly population) is $(T - Z) \cdot (N/T)$. Recall that the total population at all points in time is N. If we express the proportion of the aged population as γ, then we obtain the following equation:

$$\gamma = 1 - \frac{Z}{T}. \tag{40}$$

From Eq. (40) the proportion of the aged population, γ, depends on the life expectancy T and the retirement age Z. An increase in T or a decrease in Z dictates a rise in the proportion of the aged population. Consequently, a comparison of the respective steady-state equilibria between those before and after either parameter changes allows an understanding of the effect of the progress of ageing on the endogenous variables.

First, the connection between population ageing and the rate of long-term innovation will be examined. Partially differentiating Eq. (38) with respect to life expectancy, we obtain

$$\frac{\partial g}{\partial T} = -\frac{1}{\phi}\left\{\Big(\frac{\zeta-\alpha}{\alpha\zeta}\Big)\frac{\lambda N}{T^2}Ze^{\mu\nu(\mu,\rho,Z)}\Big[\frac{e^{-\rho\nu(\mu,\rho,Z)}-e^{-\rho Z}}{1-e^{-\rho Z}}\Big]\right\} < 0.$$

In addition, partially differentiating Eq. (38) with respect to retirement age, we obtain

$$\begin{aligned}\frac{\partial g}{\partial Z} &= \frac{1}{\phi}\Big(\frac{\zeta-\alpha}{\alpha\zeta}\Big)\frac{\lambda N}{T}e^{\mu\nu(\mu,\rho,Z)}[1+Z(\mu-\rho)]\Big[\frac{e^{-\rho\nu(\mu,\rho,Z)}-e^{-\rho Z}}{(1-e^{-\rho Z})^2}\Big]\\ &\quad+\frac{1}{\phi}\left\{\Big(\frac{\zeta-\alpha}{\alpha\zeta}\Big)\frac{\lambda N}{T}Ze^{\mu\nu(\mu,\rho,Z)}\Big[\frac{e^{-\rho\nu(\mu,\rho,Z)}-e^{-\rho Z}}{1-e^{-\rho Z}}\Big]-\rho\right\}\\ &\quad\times\Big(\frac{\zeta-\alpha}{\alpha\zeta}\Big)\frac{\lambda N}{T}(\rho Z+e^{-\rho Z}-1)\frac{[e^{-\rho\nu(\mu,\rho,Z)}-e^{-\rho Z}][Z-\nu(\mu,\rho,Z)]}{(1-e^{-\rho Z})^2[Z-\nu(\mu,\rho,Z)]^2} > 0.\end{aligned}$$

Other things being equal, the proportion of the aged population is high for economies in which the value of T is high (or the value of Z is low). Consequently, from the perspective of our model, a decline in the rate of long-term innovation is expected as the population ages.

Because an increase in Z means a higher mandatory retirement age for workers, the result, $\partial g/\partial Z > 0$, can be interpreted as an increase in the rate of long-term innovation along with a rise in the mandatory retirement age. Furthermore, a rise in the rate of innovation will contribute to an increase in national income. Thus, the model implies that encouraging employment of elderly people through an increase in the mandatory retirement age is important to achieve sustained economic growth. For many developed countries in which ageing will accelerate in the future, a rise in the mandatory retirement age will be one focal issue in arguments about employment policies. The analysis of this study can be considered as a rationale to support the policy of encouraging the employment of elderly people, as proposed by Yashiro (1997), OECD (2006), and others.

3.2 Ageing and Wage Premium

Next, we consider the direction of changes to the wage premium that accompanies the progress of ageing. The wage premium is defined as the ratio of the wage earned by a skilled worker to that of an unskilled worker. The wage rate of unskilled workers is normalised to 1 and the wage rate of skilled workers is w, and so the wage premium is $w/1 = w$. Differentiating Eq. (35) with respect to retirement age, we get

$$\frac{\partial w}{\partial Z} = \frac{e^{\mu\nu(\mu,\rho,Z)}[e^{-\rho\nu(\mu,\rho,Z)} - e^{-\rho Z}]}{\left\{[e^{-\rho\nu(\mu,\rho,Z)} - e^{\rho Z}]e^{\mu\nu(\mu,\rho,Z)}\right\}^2}[\rho - (1 - e^{-\rho Z})\mu]. \tag{41}$$

From Eq. (41), the sign of $\partial w/\partial Z$ is determined by the sign of $\rho - (1 - e^{-\rho Z})\mu$. We will investigate graphically the relations existing between population ageing and the wage premium. The curve in Figure 3 is drawn by regarding $(1 - e^{-\rho Z})\mu$ as a function of Z.

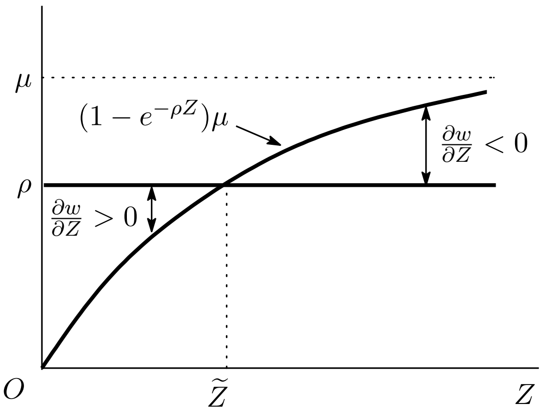

Figure 3: Relationship between Aging and Wage Premium

Figure 3 shows that if the value of Z is less than $\widetilde{Z}$ then $\partial w/\partial Z < 0$, and if it is greater than $\widetilde{Z}$ then $\partial w/\partial Z > 0$. If $Z = \widetilde{Z}$, then $\partial w/\partial Z = 0$. Therefore, in an economy in which workers' mandatory retirement age is higher than $\widetilde{Z}$, the wage premium increases as ageing progresses. In contrast, in an economy in which workers' mandatory retirement age is lower than $\widetilde{Z}$, the wage premium declines as ageing progresses. In other words, the effect of population ageing on wage premium changes at a certain age $\widetilde{Z}$. Thus, our model implies that the effect on the wage disparity between skilled and unskilled workers of promoting the employment of older workers by increasing mandatory retirement age depends on the initial mandatory retirement age.

3.3 R&D Subsidy

In this subsection we analyse the effect of R&D subsidy policy on industrial innovation. Assume a situation in which R&D subsidy policy is funded with lump sum taxes paid by households. All revenues from lump sum taxes are allocated to the subsidies for R&D firms.

Let us consider the case in which a certain percentage $\xi \in (0,1)$ of the R&D expenditures of each firm is aided by the government. The cost of R&D activities needed by firm k at time t is $w(t)H_f(t)$. Therefore, this firm receives a subsidy in the amount of $\xi w(t)H_f(t)$ from the government. Consequently, the total amount of R&D subsidies from the government at time t is $\xi w(t)H(t)$.

We denote the rate of long-term innovation when such R&D subsidies are not implemented by g^*, and the rate of long-term innovation when R&D subsidies are implemented by g^{**}. Therefore, g^* is equivalent to the rate of innovation indicated by Eq. (38). Also, it is easily seen that g^{**} is given by

$$g^{**} = \Big(\frac{1-\xi}{\phi-\xi}\Big)\Bigg\{\Big(\frac{\zeta-\alpha}{\alpha\zeta}\Big)\frac{\lambda N}{T}Ze^{\mu\nu(\mu,\rho,Z)}\frac{[e^{-\rho\nu(\mu,\rho,Z)}-e^{-\rho Z}]}{(1-\xi)(1-e^{-\rho Z})}-\rho\Bigg\}.$$

Comparing of the sizes of g^* and g^{**} reveals that $g^{**} > g^*$ holds. Consequently, R&D subsidy policy increases a firm's incentives for R&D investment and helps achieve an improved rate of innovation.

3.4 National Innovation System

We now review the determinants of innovation, in particular examining the parameter λ on the right-hand side of Eq. (38). Apparently, the greater the value of λ, the higher the rate of long-term innovation. In Eq. (17), λ was introduced as a structural parameter of the R&D production function model for new product design. The following relation holds, based on Eq. (17):

$$\frac{\partial \dot{a}_f}{\partial H_f} = \frac{\dot{a}_f}{H_f} = \lambda A.$$

Therefore, λ is considered to be a factor in the marginal or average productivity of human capital having a new product design as the output. Such a parameter of productivity can also be interpreted as an indicator of the organisational capability of an R&D firm. The organisational capability is a concept in business administration referring to the system of organisational routines that facilitates effective product design and development.

Organisational capability of firms depends not only on internal factors, but also on the institutional foundation of a national economy, which is called the social infrastructure by Hall and Jones (1999). Internal factors include knowledge management such as knowledge cordification and application functions within knowledge-creating organisations (Kakabadse, Kouzmin, and Kakabadse, 2001). For the institutional foundation of a national economy, the national innovation system may provide an important clue. Following Archibugi and Michie (2003), in this article, the national innovation system is regarded as a nation-specific factor that has a crucial role in shaping technical change.

As suggested by the empirical study of Aoki and Branstetter (2010), the importance of having constant access to the outcomes of advanced basic research has been increasing in the development of industrial technology in recent years. Based on econometric analyses using firm-level data, Aoki and Branstetter (2010) confirmed the significance to increased R&D productivity in the United States of increased knowledge diffusion from academic institutions to private-sector industries. The contribution was evident particularly in areas related to pharmaceuticals, medical equipment and biotechnology. In comparison to the United States and EU countries, however, the level of science linkage in the domestic industries of Japan is generally low.

As for institutions that aim to diffuse the outcomes of advanced basic research, university-industry collaboration constitutes an important element of the national innovation system. Interaction between universities and industries based on flexible system design may facilitate sustained economic growth through active innovation. Accordingly, since 1998, a series of measures to promote university-industry collaboration has been adopted. At present, Japan's major universities have established a technology licensing organization (TLO). On the other hand, in a macroeconomic view, the foundation of a TLO in Japan has not necessarily encouraged university-industry collaboration to function effectively and create innovation. Because university-industry collaboration is positioned, one fundamental point of the national innovation system, the method of university-industry collaboration in Japan has considerable room for improvement.

Goto and Woolgar (2006) considered university-industry-government collaboration in the United Kingdom as an example in this regard, and made remarks that offer various suggestions. Examples include a program called the industrial secondment scheme (ISS) that was adopted by the Royal Academy of Engineering. In this program, a university lecturer works in industry for 3-6 months and acquires practical experience in the applicability of their own research. Part or all the lecturer's salary during this period is paid by the Royal Academy. Another example, for transferring technology from a university to industry through participation by graduate students, is the knowledge transfer partnership (KTP) system. In this program, graduate students participate in research and development projects run jointly by the university and companies as KTP Associates, and enhance the research level in the area of science and technology relevant to industry by considering the direction of research demanded by the industrial world. Through human resource development in the KTP program, industries can expect to employ researchers who are equipped with advanced research knowledge from universities and industrial R&D expertise from their practical experience.

In a knowledge-based economy accompanied by population decrease and aging, acquiring researchers in science and technology will be increasingly important for industries. In the past, Japanese companies have not been active in hiring graduates who have a doctoral degree. However, the increase in the number of students who have acquired explicit and tacit knowledge of industrial R&D, through programs such as KTP described above, and skills in advanced research is expected to be beneficial for both universities and industries. Programs in the manner of ISS and KTP are yet to be implemented in Japan, and are therefore expected to be rather informative for innovation policy makers in Japan.

Movement of people between industry and universities has been institutionally difficult in Japan, which has limited the opportunities for a close exchange of knowledge as a result. A system of interaction between industry and academia based on flexible system design would facilitate sustainable growth through active innovation. Nonetheless, research into the national innovation system from the perspective of social science has not been adequate. Development of effective policy prescriptions requires multidisciplinary knowledge encompassing economics, management engineering, technology management, and other fields.

4 Conclusion

The underlying mechanism behind long-term growth performance associated with population ageing is yet to be adequately understood. However, very few studies in the literature have attempted to examine links between population ageing and innovation. This article is an attempt to fill that void. Specifically, a main contribution of this article is to present a manageable framework for analysing the macroeconomic effects of population ageing, extending the model of Grossman and Helpman (1991, Ch.5).

We show that the rate of technical progress will decline with population ageing, because population ageing inhibits technical progress and consequently also economic growth. In addition, we show that raising the mandatory retirement age aids technical progress. This suggests that encouraging the employment of elderly people is important for promoting economic growth. On the other hand, raising the mandatory retirement age may cause greater wage income disparities depending on the rate of return on investment in education and the rate of time preference in the economy.

How does raising the mandatory retirement age affect growth performance? Based on our model, the theoretical mechanism might be explained intuitively as follows. First, a higher retirement age increases the number of active skilled workers engaging in R&D activities. This has a direct positive effect on innovation. Second, an increase in the retirement age also increases the number of active unskilled workers. As a result, profit from the production of consumer goods increases, which generates increased R&D revenue to promote product innovation. Third, raising the retirement age means a longer period of time during which skilled workers are able to earn income. Thus, the incentives for economic agents to gain skills through education and training will increase. As a result, there is an increased accumulation of human capital in the macro-economy that will increase the rate of innovation and, therefore, national income.

Despite our simplifying assumptions, our model captures salient features of the real world. We hope that it will serve as a stepping-stone towards developing future work.

References

[1] Aoki, R. and L. Branstetter (2010), "Is Academic Science Raising Innovative Productivity? Theory and Evidence from Firm-Level Data," In Y. Kitamura (ed.), *Microeconometrics*, Nippon Hyoronsha, Tokyo (in Japanese).

[2] Archibugi, D. and J. Michie (2003), "Technological Globalization of National Systems of Innovation?," In L. Paganetto and E. S. Phelps (eds.), *Finance, Research, Education and Growth*, Palgrave, New York.

[3] Barro, R. J. and X. Sala-i-Martin (2004), *Economic Growth*, Second Edition, The MIT Press, Cambridge, Massachusetts.

[4] Futagami, K. and T. Nakajima (2001), "Population Aging and Economic Growth," *Journal of Macroeconomics*, vol. 23, pp. 31-44.

[5] Futagami, K., T. Iwaisako, and T. Nakajima (2002), "Technological Innovation, Productivity, and Economic Growth in Aging Societies," In H. Siebertin (ed.), *Economic Policy for Aging Societies*, Springer-Verlag, Berlin.

[6] Goto, A. and L. R. Woolgar (2006), "University-Industry-Government Collaboration in the Development of Human Resources for Science and Technology," In A. Goto and T. Kodama (eds.), *Japan's National Innovation System: Rebuilding the Engine of Growth*, University of Tokyo Press, Tokyo (in Japanese).

[7] Grossman, G. M. and E. Helpman (1991), *Innovation and Growth in the Global Economy*, The MIT Press, Cambridge.

[8] Gruescu, S. (2007), *Population Ageing and Economic Growth: Education Policy and Family Policy in a Model of Endogenous Growth*, Physica-Verlag, Heidelberg.

[9] Hall, R. E. and C. I. Jones (1999), "Why Do Some Countries Produce So Much More Output per Worker than Others?," *Quarterly Journal of Economics*, vol. 114, pp. 83-116.

[10] Jones, C. I. (2002), *Introduction to Economic Growth,* Second Edition, W. W. Norton, New York.

[11] Kakabadse, N. K, A. Kouzmin, and A. Kakabadse (2001), "From Tacit Knowledge to Knowledge Management: Leveraging Invisible," *Knowledge and Process Management*, vol. 8, pp. 137-154.

[12] Lee, J-W. (2005) "Human Capital and Productivity for Korea's Sustained Economic Growth," *Journal of Asian Economics*, vol. 16, pp. 663-687.

[13] Lucas, R. E. Jr. (1988) "On the Mechanics of Economic Development," *Journal of Monetary Economics*, vol. 22, pp. 3-42.

[14] Odagiri, H. (2001), *Modern Industrial Organization: Theoretical and Empirical Approaches and Competition Policy*, Yuhikaku, Tokyo (in Japanese).

[15] Organisation for Economic Co-operation and Development (2001), *Education Policy Analysis 2001*, OECD, Paris.

[16] Organisation for Economic Co-operation and Development (2006), *Live Longer, Work Longer: Ageing and Employment Policies*, OECD, Paris.

[17] Psarras, J. E. (2007), "Education and Training in the Knowledge-based Economy: The Application of Knowledge Management," *International Journal of Information Technology and Management*, vol. 6, pp. 92-104.

[18] Romer, P. M. (1990) "Endogenous Technological Change," *Journal of Political Economy*, vol. 98, pp. S71-S102.

[19] Yashiro, N. (1997), "Aging of the Population in Japan and Its Implications to the Other Asian Countries," *Journal of Asian Economics,* vol. 8, pp. 245-261.