Bayesian Estimation of the CES Production Function with Labor- and Capital-Augmenting Technical Change

Hideo Noda* and Koki Kyo†

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* Yamagata University
† Obihiro University of Agriculture and Veterinary Medicine
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Abstract

In this paper, we propose a new approach to time-series analysis of factor augmenting technical change based on a constant elasticity of substitution (CES) production function. To estimate trends in labor- and capital-augmenting technical change, smoothness priors are introduced and Bayesian linear models are constructed. Parameter estimations are then calculated using the methods of maximum likelihood and Bayesian model averaging. As a practical application, we examine the technical changes in Taiwan and South Korea at the macroeconomic level. The results of this analysis illustrate that our Bayesian approach can capture the movements of technical change rigorously compared with conventional approaches.

Keywords: Factor augmenting technical progress; CES production functions; Smoothness priors; Bayesian linear modeling; Bayesian model averaging

JEL classification: C11; C22; O30

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†Corresponding author. Faculty of Literature and Social Sciences, Yamagata University; 1-4-12 Kojirakawa-machi, Yamagata 990-8560, Japan; (e-mail): noda@human.kj.yamagata-u.ac.jp

‡Department of Agro-Environmental Science, Obihiro University of Agriculture and Veterinary Medicine; Inada-cho, Obihiro, Hokkaido 080-8555, Japan; (e-mail): kyo@obihiro.ac.jp
1 Introduction

Understanding trends in technical change has significant implications when forecasting long-term economic performance. A measure of technical change is seen as a key issue in empirical studies of economic growth. However, even if we can define technical progress within the framework of a theoretical analysis, it is very difficult to quantify that actual aspect. Therefore, models and statistical methods for estimating factors that reflect technical progress become indispensable tools. The aim of this paper is to develop a Bayesian approach to time-series analysis for labor- and capital-augmenting technical change.

The starting point for this type of econometric analysis is the construction of a framework based on production functions. Many researchers use the Cobb-Douglas production function or the CES production function with Hicks neutral technical change. It should be noted, however, that there are some drawbacks with these. For example, although the Cobb-Douglas production function assumes an elasticity of substitution equal to one, its true value in an economy is not necessarily very close to unity. Also, observations that the shares of the labor income and the capital income change in an almost constant manner during a certain period, are often considered as evidence supporting the use of the Cobb-Douglas production function. However, as pointed out by Sato (1970), the invariance of each share does not guarantee that the functional form is consistent with Cobb-Douglas type. As for the CES production function with Hicks neutral technical change, the implication is that the efficiencies in production of labor and capital are identical. This is a very strong assumption, and obviously unusual: the statistical analyses based on such CES production functions have a narrow focus and scope. Hence, any discussion based on such analysis becomes extremely limited.

Considering these problems in statistical analyses that can be seen in many previous studies, we adopt a CES production function with factor-augmenting (biased) technical change. The term biased technical change refers to a situation in which the efficiency of capital and the efficiency of labor exhibit different growth rates. Hence, movements in the capital- and labor-augmenting technical progress can be clearly identified.

Recent studies related to the present paper include Antràs (2004), Klump et al. (2007b) and Sato and Morita (2009).\(^1\) Antràs (2004) and Klump et al. (2007b) examined the econometric estimations of production functions using data stemming from the US economy. Sato and Morita (2009) estimated the growth rates of capital and labor efficiencies for the Japanese and US economies according to equations

\(^1\)See Klump et al. (2007a) for a review article on empirical studies of CES production functions.
that are derived in Sato (1970). These articles treat CES production functions with technical change that augment the efficiency of both capital and labor, and present careful analyses. Nevertheless, we consider that there is still room for improvement. For specifications for the trends in technical change, Antràs (2004) follows the bulk of the literature in assuming that the efficiencies in both labor and capital grow at constant rates.\(^2\) Also, Klump \textit{et al.} (2007b) assumes constant growth rates as well as a form that includes exponential, logarithmic and hyperbolic growth as special cases. It seems to give rise to inflexible models because the actual technical progress might not necessarily show the trends that Antràs (2004) and Klump \textit{et al.} (2007b) assume. Moreover, even if the fit of the model is suitable for the relevant period, the forecast might be wide of the mark when the assumed trends are not forthcoming. For an estimation of the factor-augmenting technical changes, Sato and Morita (2009) did not \textit{a priori} assume exponential, hyperbolic, or any other form. In that sense, the method of Sato and Morita (2009) might be superior to those of Antràs (2004) and Klump \textit{et al.} (2007b). However, we think that problems will arise as their proposed method cannot calculate the elasticity of factor substitution directly from a CES production function with factor-augmenting technical change. To estimate the capital- and labor-augmenting technical changes, they use the elasticity of substitution derived from a CES production function with Hicks neutral technical change as a proxy.

Unlike Antràs (2004), Klump \textit{et al.} (2007b) and Sato and Morita (2009), we address the estimation of this factor-augmenting technical change from a Bayesian perspective. Specifically, we combine the use of a Bayesian linear modeling approach in Akaike (1980) and a smoothness-priors approach in Kitagawa and Gersch (1996). By introducing the smoothness priors, we can set up the model with a sufficiently flexible structure to capture the dynamic features of the economic system, hence our model is very useful for estimating complicated behaviors of economic variables that cannot be observed directly. Studies on the application of such an approach to the analyses of dynamic structure of an economy are rare. The exceptions include Kyo and Noda (2008) and Noda and Kyo (2009a, 2009b). In Kyo and Noda (2008), smoothness priors were incorporated into a statistical model based on a Cobb-Douglas production function, and addressed, by applying Bayesian techniques, the estimation of trends in China’s total factor productivity (TFP). Noda and Kyo (2009a) also incorporated a smoothness prior in a statistical model based on a Cobb-Douglas form for the production function, and estimated the trends in TFP for the Japanese prefectural economies. Based on a CES production function

with Hicks neutral technical change, Noda and Kyo (2009b) applied the smoothness priors approach in a comparative analysis of the technical change in the Japanese and U.S. macro-economies. Recall, however, that there are some drawbacks mentioned above in the Cobb-Douglas production functions and the CES production functions with Hicks neutral technical change.

In our framework, the efficiencies of capital and labor are regarded as time-varying parameters, and thereby introduce dynamic features into capital- and labor-augmenting technical change. Parameter estimations are then carried out using the methods of maximum likelihood and Bayesian model averaging. As a practical application of the proposed Bayesian approach, we attempt to estimate the technical changes in the macro-economies of Taiwan and South Korea. The results of this analysis illustrate that our Bayesian approach can treat movements in technical change in more detail compared with the conventional production function approach, and therefore provide a useful insight into the empirical study of technical change.

The rest of the paper is organized as follows. In Section 2, we construct an analytical framework based on Bayesian models via a CES production function with factor augmenting technical change. Section 3 presents the basic scheme for parameter estimation and a procedure to apply Bayesian model averaging. In Section 4, our proposed approach is applied to time-series analysis of the macro economies in South Korea and Taiwan. Finally, Section 5 concludes the paper.

2 Bayesian Modeling based on a CES Production Function

Consider an aggregative economy in which each year a single homogeneous good is produced by two factors of production, capital and labor, under constant returns to scale technology. We follow David and van de Klundert (1965) by writing the production function

\[ Q_t = \left[ (A_t K_t)^{-\rho} + (B_t L_t)^{-\rho} \right]^{-\frac{1}{\rho}}, \]  

where the subscript \( t \) indexes yearly increments, \( Q_t \) the value added, \( K_t \) capital, and \( L_t \) labor. Note that \( K_t \) and \( L_t \) are stocks, that is, amounts observed at one point in time, while \( Q_t \) is a flow. The model in Eq. (1) is a specification of the production function with factor-augmenting technical change.\(^3\) The elasticity of factor substitution equals \( 1/(1 + \rho) \), hence \( \rho \) is termed the substitution parameter. The coefficients \( A_t \) and \( B_t \) indicate the efficiency in production of capital and labor, respectively.

\(^3\)See Sato and Beckmann (1968) for a derivation of the generalized form of such production functions.
In addition, variation in $A_t$ with time is interpreted as capital-augmenting technical change and variation in $B_t$ over time is interpreted as labor-augmenting technical change.

We can rewrite Eq.(1) by setting $a_t = A_t^{-\rho}$, $b_t = B_t^{-\rho}$, $x_{1t} = (-1/\rho)K_t^{-\rho}$, $x_{2t} = (-1/\rho)L_t^{-\rho}$ and $y_t = (-1/\rho)Q_t^{-\rho}$, in which notation Eq.(1) becomes

$$y_t = a_t x_{1t} + b_t x_{2t}.$$  

(2)

Furthermore, we augment Eq. (2) to include a random disturbance:

$$y_t = a_t x_{1t} + b_t x_{2t} + \epsilon_t,$$  

(3)

where $\epsilon_t$ is taken to be Gaussian white noise.

Noda and Kyo (2009b) applied a Bayesian approach using smoothness priors of Kitagawa and Gersch (1996) to estimate the Hicks neutral technical change. This Bayesian approach can also be applied in estimating the time-varying parameters $a_t$ and $b_t$. However, it may be difficult to obtain meaningful estimates for $a_t$ and $b_t$ simultaneously, because there is a remarkable multicollinearity between the time-series data for $x_{1t}$ and $x_{2t}$.

To ameliorate this difficulty, we simplify the model in Eq. (3) by treating $b_t$ as a constant $\tilde{b}$, i.e., we put $b_1 = b_2 = \cdots = \tilde{b}$. Then, the model reduces to the following form:

$$y_t = a_t x_{1t} + \tilde{b} x_{2t} + \epsilon^{(1)}_t,$$  

(4)

where $\tilde{b}$ is an unknown parameter, and $\epsilon^{(1)}_t$ is a Gaussian white noise with $\epsilon^{(1)}_t \sim N(0,\sigma^2)$. For the model in Eq. (4), the time-varying parameter $a_t$ is treated as a random variable from a Bayesian viewpoint. Thus, we use the following smoothness prior:

$$a_t - 2a_{t-1} + a_{t-2} = \nu_{1t},$$  

(5)

where $\nu_{1t}$ is a Gaussian white noise with $\nu_{1t} \sim N(0,\sigma^2/d_1^2)$, and $d_1 > 0$ is an unknown parameter. It is also assumed that $\epsilon^{(1)}_t$ and $\nu_{1t}$ are independent of each other. To define a proper prior distribution for the $a_t$’s it is necessary to set $a_0$ and $a_{-1}$. Here, all of the parameters $\rho$, $\tilde{b}$, $\sigma^2$, $a_0$, $a_{-1}$ and $d_1$ are considered as unknown constants.

When the values of these parameters are given, a system of linear Bayesian equations for $a_t$’s can be constructed based on Eqs. (4) and (5). From this, we can obtain a posterior distribution for the $a_t$’s by using the Bayesian linear modeling method.

Alternatively, in an analogous manner, the model in Eq. (3) can also be simplified by treating $a_t$ as constant, i.e., we assume that $a_1 = a_2 = \cdots = \tilde{a}$. Thus, the model
becomes the following form:

$$y_t = \tilde{a}x_{1t} + b_t x_{2t} + \epsilon^{(2)}_t,$$

where $\tilde{a}$ is an unknown parameter, $\epsilon^{(2)}_t$ is a Gaussian white noise with $\epsilon^{(2)}_t \sim N(0, \sigma^2)$. Here we treat $b_t$ as a random variable and use the smoothness prior as follows:

$$b_t - 2b_{t-1} + b_{t-2} = \nu_2 t,$$

where $\nu_2 t$ is a Gaussian white noise with $\nu_2 t \sim N(0, \sigma^2 / d_2^2)$, and $d_2 > 0$ is an unknown parameter. Again, we assume that $\epsilon^{(2)}_t$ and $\nu_2 t$ are independent of each other, and that the parameters $\rho$, $\tilde{a}$, $\sigma^2$, $b_0$, $b_{-1}$ and $d_2$ are unknown constants. When the values of these parameters are given, a system of Bayesian linear equations for the $b_t$'s can be constructed based on Eqs. (6) and (7). Then we can obtain a posterior distribution for $b_t$'s by using the Bayesian linear modeling method.

Thus, we have two similar Bayesian models that simplify the model of Eq. (3), specifically, those defined by the pair Eqs. (4) and (5) and the pair Eqs. (6) and (7). By combining the estimates from the two Bayesian models and using the Bayesian model averaging (BMA) approach, we can obtain synthetic estimates for the model in Eq. (3). Note that $\rho$ and $\sigma^2$ are parameters common to the two models.

3 Parameter Estimation

3.1 Basic Scheme

We now present a procedure for estimating the time-varying parameter $a_t$, based on the constant-$b$ Bayesian model, Eqs. (4) and (5). Assume that we have a data set for $Q_t$, $K_t$ and $L_t$ for a period of $n$ points as $t = 1, 2, \ldots, n$. From Eq. (4), the likelihood of $a = (a_1, a_2, \ldots, a_n)^t$ and the other related parameters is given by:

$$f(Q|a, \rho, \tilde{a}, \sigma^2) = J \left( \frac{1}{\sqrt{2\pi\sigma^2}} \right)^n \exp \left[ -\frac{1}{2\sigma^2} \| y - X a - \tilde{b} x \|^2 \right],$$

where $J$ is the Jacobian of the transformation from $Q$ to $y$ defined by $J = \Pi_{t=1}^n (\partial y_t / \partial Q_t)$. The other symbols are defined as follows:

$$Q = (Q_1, Q_2, \ldots, Q_n)^t,$$
$$y = (y_1, y_2, \ldots, y_n)^t,$$
$$X = \text{diag}(x_{11}, x_{12}, \ldots, x_{1n}),$$
$$x = (x_{21}, x_{22}, \ldots, x_{2n})^t,$$

and $\| * \|$ denotes the Euclidean norm.
When the values of $\sigma^2$, $c = (a_1, a_0)^t$, and $d_1$ are given, from Eq. (5) the prior density for $a$ is given by:

$$
\pi(a|\sigma^2, c, d_1) = \left(\frac{d_1}{\sqrt{2\pi \sigma^2}}\right)^n \exp \left[ -\frac{d_1^2}{2\sigma^2}||Da + Bc||^2 \right],
$$

where

$$
D = \begin{bmatrix}
1 & 0 & \cdots & \cdots & \cdots & 0 \\
-2 & 1 & \ddots & & & \\
1 & -2 & 1 & \ddots & & \\
0 & \ddots & \ddots & \ddots & \ddots & \\
\vdots & & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & 1 & -2 & 1
\end{bmatrix}, \\
B = \begin{bmatrix}
1 & -2 \\
0 & 1 \\
0 & 0 \\
\vdots & \\
0 & 0
\end{bmatrix}.
$$

Next, we obtain the marginal likelihood of the related parameters as

$$
L(\rho, \tilde{b}, \sigma^2, c, d_1) = \int \cdots \int f(Q|a, \rho, \tilde{b}, \sigma^2)\pi(a|\sigma^2, c, d_1)da
$$

$$
= j \left(\frac{1}{\sqrt{2\pi \sigma^2}}\right)^n d_1^n \det(W^tW)^{-\frac{1}{2}} \exp \left[ -\frac{1}{2\sigma^2}||h - W\tilde{a}||^2 \right],
$$

where

$$
W = \begin{bmatrix}
X \\
-\tilde{d}_1D
\end{bmatrix}, \\
h = \begin{bmatrix}
y + \tilde{b}x \\
d_1Bc
\end{bmatrix},
$$

and $\tilde{a} = (W^tW)^{-1}W^th$ denotes the mean of a posterior density

$$
f(a|Q; \rho, \tilde{b}, \sigma^2, c, d_1) = \frac{f(Q|a, \rho, \tilde{b}, \sigma^2)\pi(a|\sigma^2, c, d_1)}{\int \cdots \int f(Q|a, \rho, \tilde{b}, \sigma^2)\pi(a|\sigma^2, c, d_1)da}.
$$

for $a$. Therefore, the estimates, $\hat{\rho}$, $\hat{\tilde{b}}$, $\hat{\sigma}^2$, $\hat{c}$, and $\hat{d}_1$ for the related parameters can be obtained by maximizing the likelihood $L(\rho, \tilde{b}, \sigma^2, c, d_1)$ in Eq. (8).

Computationally, if the values of $\rho$ and $d_1$ are given, then $a$ together with $c$ and $\tilde{b}$ can be estimated simultaneously by using the least squares method (see for example, Jiang, 1995). That is, when we put

$$
z = \begin{bmatrix}
y \\
0_n
\end{bmatrix}, \\
V = \begin{bmatrix}
X & O & x \\
d_1D & d_1B & 0_n
\end{bmatrix}, \\
\beta = \begin{bmatrix}
a \\
c \\
\tilde{b}
\end{bmatrix}
$$

with $O$ denoting a zero matrix, and $0_n$ being an $n$-dimensional vector of zeros, the estimate,

$$
\hat{\beta}(\rho, d_1) = \left(\hat{a}^t(\rho, d_1), \hat{c}^t(\rho, d_1), \hat{\tilde{b}}(\rho, d_1)\right)^t
$$

$$
= (V^tV)^{-1}V^tz,
$$

7
for $\beta$ is obtained. Thus, by substituting $\tilde{b} = \tilde{b}(\rho, d_1)$ and $c = \tilde{c}(\rho, d_1)$ into $L(\rho, \tilde{b}, \sigma^2, c, d_1)$ in Eq. (8), the partial likelihood of $\rho$, $\sigma^2$ and $d_1$ can be obtained as

\[
L_1(\rho, \sigma^2, d_1) = L(\rho, \tilde{b}, \sigma^2, \tilde{c}, d_1).
\]

(10)

Correspondingly, a conditional posterior density, $g(a|Q; \rho, \sigma^2, d_1)$, for $a$ can be derived from $f(a|Q; \rho, \tilde{b}, \sigma^2, c, d_1)$ in Eq. (9) by:

\[
g(a|Q; \rho, \sigma^2, d_1) = f(a|Q; \rho, \tilde{b}(\rho, d_1), \sigma^2, \tilde{c}(\rho, d_1), d_1).
\]

Moreover, the estimate $\hat{d}_1$ of the parameter $d_1$ is obtained by maximizing the marginal likelihood $L_1(\rho, \sigma^2, d_1)$ in Eq. (10) for given $\rho$ and $\sigma^2$. Thus, we obtain the partial likelihood of $\rho$ and $\sigma^2$ as

\[
L^*_1(\rho, \sigma^2) = L_1(\rho, \sigma^2, \hat{d}_1).
\]

(11)

Note that the procedure for estimating the parameter $b = (b_1, b_2, \ldots, b_n)^t$ in the constant-$\beta$ Bayesian model, Eqs. (6) and (7), is the same, and is omitted here for brevity. For this model, we can obtain the partial likelihood $L_2(\rho, \sigma^2, d_2)$ of $\rho$, $\sigma^2$ and $d_2$ by a procedure similar to the above basic scheme. Then the partial likelihood of $\rho$ and $\sigma^2$ can be obtained by using the maximum likelihood estimate $\hat{d}_2$ of $d_2$ as

\[
L^*_2(\rho, \sigma^2) = L_2(\rho, \sigma^2, \hat{d}_2).
\]

(12)

### 3.2 Bayesian Model Averaging Approach

To obtain the estimates for $\rho$ and $\sigma^2$ and the final estimates for parameters $a$ and $b$ of the model in Eq. (3), we use the BMA approach as follows. Here, we reconsider the two Bayesian models with constant-$b$ and constant-$a$; these models are identified here as $M_1$ and $M_2$ respectively. From the BMA perspective (see for example, Claeskens and Hjort, 2008), it is assumed that all models $\{M_1, M_2\}$ are uncertain and all have equal uncertainties. Thus, we use a uniform prior for the models $\{M_1, M_2\}$ as follows:

\[
q(M_1) = q(M_2) = \frac{1}{2},
\]

where $q(M_i)$ is the prior probability for the $i$-th model $M_i$ $(i = 1, 2)$. As shown in the preceding subsection, a partial likelihood for the model $M_i$ is $L^*_i(\rho, \sigma^2)$, as given in Eqs. (11) and (12). Therefore, a conditional posterior probability for model $M_i$ is given by

\[
p(M_i|Q; \rho, \sigma^2) = \frac{L^*_i(\rho, \sigma^2)q(M_i)}{L^*_1(\rho, \sigma^2)q(M_1) + L^*_2(\rho, \sigma^2)q(M_2)}
\]

\[= \frac{L^*_i(\rho, \sigma^2)}{L^*_1(\rho, \sigma^2) + L^*_2(\rho, \sigma^2)}.
\]
Moreover, from the basic estimation scheme just mentioned, we can see that for model \( M_1 \) given \( \rho \) and \( d_1 \), the posterior mean of \( a \) is \( \hat{a}(\rho, d_1) \) and the estimate of \( b = \hat{b}_1 \) is \( \hat{b}(\rho, d_1) \). Similarly, for model \( M_2 \) given \( \rho \) and \( d_2 \), the posterior mean of \( b \) is \( \hat{b}(\rho, d_2) \) and the estimate of \( a = \hat{a}_1 \) is \( \hat{a}(\rho, d_2) \). Therefore, based on the Bayesian model averaging approach, the synthetic estimates for \( a \) and \( b \) are given by (see, Draper, 1995):

\[
\begin{align*}
\hat{a}(\rho, \sigma^2) &= p(M_1|Q, \rho, \sigma^2)\hat{a}(\rho, \hat{d}_1) + p(M_2|Q, \rho, \sigma^2)\hat{a}(\rho, \hat{d}_2) \mathbf{1}_n, \\
\hat{b}(\rho, \sigma^2) &= p(M_2|Q, \rho, \sigma^2)\hat{b}(\rho, \hat{d}_2) + p(M_1|Q, \rho, \sigma^2)\hat{b}(\rho, \hat{d}_1) \mathbf{1}_n.
\end{align*}
\]

Furthermore, we consider the quantity

\[
\mathcal{L}(\rho, \sigma^2) = \mathcal{L}^*_1(\rho, \sigma^2)q(M_1) + \mathcal{L}^*_2(\rho, \sigma^2)q(M_2)
\]

\[
= \frac{1}{2} \left( \mathcal{L}^*_1(\rho, \sigma^2) + \mathcal{L}^*_2(\rho, \sigma^2) \right)
\]  

(13)

to be a measure of the fitting reliability for the model in Eq. (3) with respect to the parameters \( \rho \) and \( \sigma^2 \). Thus, the estimates \( \hat{\rho} \) and \( \hat{\sigma}^2 \) of \( \rho \) and \( \sigma^2 \) can be obtained by maximizing the mean likelihood \( \mathcal{L}(\rho, \sigma^2) \) in Eq. (13) with respect to these parameters. Hence, the final estimates for \( a \) and \( b \) are obtained as

\[
\hat{a} = \hat{a}(\hat{\rho}, \hat{\sigma}^2), \quad \hat{b} = \hat{b}(\hat{\rho}, \hat{\sigma}^2).
\]

4 Application

4.1 Data Sources

To estimate the CES production function for Taiwan, we use real GDP \((Q_t)\) and non-land fixed physical capital \((K_t)\) from Chow and Lin (2002).\(^4\) In addition, labor data \((L_t)\) are calculated according to the product of the number of employed workers from Mizoguchi (2008) and the quality of human capital per worker from Chow and Lin (2002). The annual time-series data for Taiwan cover the period from 1951 to 1999.

Also, we conducted parameter estimations of the CES production function for South Korea using macro-data taken from Pyo (2001), which are real GDP \((Q_t)\), physical capital \((K_t)\), and the number of employed workers \((L_t)\). The annual time-series data for South Korea cover a similar period, from 1946 to 1999.

---

\(^4\)The non-land fixed capital comprises fixed assets excluding land values. Chow and Lin (2002) excluded land values and cumulative depreciation from total gross fixed assets to obtain a net value for non-land fixed assets.
4.2 Results and Discussion

When we calculated the estimation procedure presented in Section 3, we used yearly data with values from each year divided by the value in the initial year, that is, \( \frac{Q_t}{Q_1}, \frac{K_t}{K_1} \) and \( \frac{L_t}{L_1} \) for \( t = 1, 2, \ldots, n \), where \( t = 1 \) denotes the initial year and \( t = n \) denotes the final year in our sample. Considering this data, we follow de La Grandville (2009) by rewriting Eq. (1) as

\[
\frac{Q_t}{Q_1} = \left[ \delta \left( h_t \frac{K_t}{K_1} \right)^{-\rho} + (1-\delta) \left( g_t \frac{L_t}{L_1} \right)^{-\rho} \right]^{-\frac{1}{\rho}}. \tag{14}
\]

In Eq. (14), we call \( h_t \) and \( g_t \) the modified efficiencies in production of capital and labor, respectively. Under the assumptions \( h_1 = g_1 = 1 \) and competitive factor imputation, \( \delta \) denotes the share of capital income from the total income at base year \( t = 1 \) and \( (1-\delta) \) denotes the corresponding share of labor income (see, de La Grandville, 2009 for details). Note that we have relations \( a_t = A_t^{-\rho} = \delta h_t^{-\rho} \) and \( b_t = B_t^{-\rho} = (1-\delta) g_t^{-\rho} \).

As mentioned earlier, the CES production function with Hicks neutral technical change is a special case of the CES production function with factor augmenting technical change. To confirm the superiority of the factor augmenting type as a statistical model, we estimated not only the CES production function with factor augmenting technical change in the present paper, but also the CES production function with Hicks neutral technical change, such as Noda and Kyo (2009b) using the data set mentioned above. As a result, when we use the Taiwan data, the value of the Akaike information criterion (AIC), given Hicks neutral type is \(-17.052\), while that for the factor augmenting type is \(-398.025\). Also, when we use the South Korea data, the value of AIC, given Hicks neutral type is \(-0.017\), while that for the factor augmenting type is \(-367.431\). Consequently, from the viewpoint of the minimum AIC model selection procedure, it is obvious that the factor augmenting type is better model. Thus, we present only the results of the estimation for the CES production function with factor augmenting technical change.

For Taiwan, the estimated results of the constant parameters are as follows: \( \hat{d}_1 = 3.4924, \hat{d}_2 = 4.4152, \hat{\rho} = -0.09418 \), and \( \hat{\sigma}^2 = 4.0896 \times 10^{-5} \). For South Korea, these are: \( \hat{d}_1 = 10.7064, \hat{d}_2 = 14.3917, \hat{\rho} = -0.1504 \), and \( \hat{\sigma}^2 = 4.6191 \times 10^{-4} \). The estimate for the elasticity of substitution can be calculated from the estimate \( \hat{\rho} \). Specifically, these estimates for Taiwan and South Korea are 1.104 and 1.177, respectively. De La Grandville (2009) derived and proved two remarkable theorems on the relationship between the elasticity of substitution and performance of macro-economy. These can be stated as follows:
(i) If an economy is driven by a CES production function exhibiting capital- and labor-augmenting technical progress, then at any stage of its development per capita income is an increasing function of its elasticity of substitution.

(ii) If an economy is driven by a CES production function exhibiting capital- and labor-augmenting technical progress, then at any stage of its development the cost of producing one unit of national product is a decreasing function of the elasticity of substitution.

According to these two theorems, we can conjecture that the long-term per capita income in South Korea may be greater than that in Taiwan, and the long-term cost of producing one unit national product in South Korea may be lower than in Taiwan.

Tables 1 and 2 show the estimates \( \hat{a}_t \) and \( \hat{b}_t \) for Taiwan and South Korea respectively.

**Table 1: Estimated results of the time-varying parameters for Taiwan**

<table>
<thead>
<tr>
<th>( t )</th>
<th>( \hat{a}_t )</th>
<th>( \hat{a}_t )</th>
<th>( \hat{b}_t )</th>
<th>( \hat{b}_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1951</td>
<td>0.69462</td>
<td>1976</td>
<td>0.72444</td>
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Table 2: Estimated results of the time-varying parameters for South Korea

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Because, by definition, $a_t = A_t^{-\rho} = \delta h_t^{-\rho}$ and $b_t = B_t^{-\rho} = (1 - \delta)g_t^{-\rho}$, we obtain $a_t^{-1/\rho} = A_t = \delta^{-1/\rho}h_t$ and $b_t^{-1/\rho} = B_t = (1 - \delta)^{1/\rho}g_t$. Furthermore, the relations $A_t/A_1 = h_t/h_1$ and $B_t/B_1 = g_t/g_1$ hold. In other words, the index of efficiency of capital ($A_t/A_1$) synchronizes with the index of the modified efficiency of capital ($h_t/h_1$). A similar remark holds between ($B_t/B_1$) and ($g_t/g_1$). We can obtain the estimates $\hat{A}_t$ and $\hat{B}_t$ from $\hat{a}_t^{-1/\rho}$ and $\hat{b}_t^{-1/\rho}$, respectively.
Figure 1 presents graphs for \((\hat{A}_t/\hat{A}_1) \times 100\) and \((\hat{B}_t/\hat{B}_1) \times 100\) for Taiwan.

From Figure 1, we find clearly that the efficiencies of labor and capital in Taiwan have changed at different growth rates. In particular, the rapid growth in labor efficiency is remarkable. This result suggests that labor-augmenting technical progress had contributed much to Taiwan’s economic growth during the five-decade period.

Figure 2 shows the graphs of \((\hat{A}_t/\hat{A}_1) \times 100\) and \((\hat{B}_t/\hat{B}_1) \times 100\) for South Korea.

The following features are seen from Figure 2. First, movements in labor and capital efficiencies for South Korea are more complex than those of Taiwan. Moreover, we find a similar pattern between the efficiencies, although the curves differ in magnitude. An upward tendency is seen in both over the period from the 1980s to the early 1990s. However, this movement stagnated during the late 1990s.
The following conclusions are obtained from a comprehensive viewpoint. First, there is a possibility that capital efficiency and labor efficiency have distinctly different trends. Figure 1 for Taiwan is one such example. Moreover, as shown in Figure 2 for South Korea, movements of efficiencies of capital and labor may be non-monotonic. Thus, our practical application suggests that modeling with a high degree of flexibility is indispensable in capturing the trends in technical change. In other words, the model should be able to deal with the diversity and complexity in the behavior of technical change. However, many earlier studies impose stringent constraints on the trends in efficiencies of capital and labor, such as the efficiencies being exponentially dependent on time. Thus, conventional approaches that lack the necessary flexibility within the model cannot estimate trends in technical change precisely, and might be misleading.

5 Conclusion

Recently, the underlying role of technical progress in economic growth has become increasingly important. However, statistical methods for estimating technical change have so far not been sufficiently developed. We improve the conventional production function based approaches by using Bayesian modeling, and propose a new methodology in the time-series analysis of capital- and labor-augmenting technical change.

As an illustration, we applied our proposed approach to an analysis of data obtained for Taiwan and South Korea. Our main results can be summarized as follows: The Taiwanese economy experienced rapid growth in labor efficiency from 1950s to 1990s, while capital efficiency showed a modest growth during the same period. This implies that the labor-augmenting technical progress had contributed much to Taiwan’s economic growth over the period. For South Korea, a similar pattern was seen for labor and capital efficiencies, although each developed at different levels and growth rates. In particular, both had shown an upward tendency from the 1980s to the early 1990s, but movements stagnated during the late 1990s. Our estimations suggest that to sustain future economic growth in South Korea, policies need to be established to promote both capital- and labor-augmenting technical progress.

To our knowledge, attempts at analyzing concepts like technical progress as in the present study have not yet been reported. The Bayesian perspective as contributed here sheds light on an approach to capture such concepts as the factor augmenting technical change. In particular, because our approach enables a statistical analysis of trends in technical change in a rigorous manner by a flexible structure of the models introducing smoothness priors, it has definite superiority over conventional
approaches. Therefore, our approach can be regarded as a promising tool for an empirical study of technical change.

References


