Bayesian Estimation of the CES Production Function with Labor- and Capital-Augmenting Technical Change

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Abstract

In this paper, we propose a new approach to time-series analysis of factor augmenting technical change based on a constant elasticity of substitution (CES) production function. To estimate trends in labor- and capital-augmenting technical change, smoothness priors are introduced and Bayesian linear models are constructed. Parameter estimations are then calculated using the methods of maximum likelihood and Bayesian model averaging. As a practical application, we examine the technical changes in Taiwan and South Korea at the macroeconomic level. The results of this analysis illustrate that our Bayesian approach can capture the movements of technical change rigorously compared with conventional approaches.

Keywords: Factor augmenting technical progress; CES production functions; Smoothness priors; Bayesian linear modeling; Bayesian model averaging

JEL classification: C11; C22; O30

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1 Introduction

Understanding trends in technical change has significant implications when forecasting long-term economic performance. A measure of technical change is seen as a key issue in empirical studies of economic growth. However, even if we can define technical progress within the framework of a theoretical analysis, it is very difficult to quantify that actual aspect. Therefore, models and statistical methods for estimating factors that reflect technical progress become indispensable tools. The aim of this paper is to develop a Bayesian approach to time-series analysis for labor- and capital-augmenting technical change.

The starting point for this type of econometric analysis is the construction of a framework based on production functions. Many researchers use the Cobb-Douglas production function or the CES production function with Hicks neutral technical change. It should be noted, however, that there are some drawbacks with these. For example, although the Cobb-Douglas production function assumes an elasticity of substitution equal to one, its true value in an economy is not necessarily very close to unity. Also, observations that the shares of the labor income and the capital income change in an almost constant manner during a certain period, are often considered as evidence supporting the use of the Cobb-Douglas production function. However, as pointed out by Sato (1970), the invariance of each share does not guarantee that the functional form is consistent with Cobb-Douglas type. As for the CES production function with Hicks neutral technical change, the implication is that the efficiencies in production of labor and capital are identical. This is a very strong assumption, and obviously unusual: the statistical analyses based on such CES production functions have a narrow focus and scope. Hence, any discussion based on such analysis becomes extremely limited.

Considering these problems in statistical analyses that can be seen in many previous studies, we adopt a CES production function with factor-augmenting (biased) technical change. The term biased technical change refers to a situation in which the efficiency of capital and the efficiency of labor exhibit different growth rates. Hence, movements in the capital- and labor-augmenting technical progress can be clearly identified.

Recent studies related to the present paper include Antràs (2004), Klump *et al.* (2007b) and Sato and Morita (2009).[1] Antràs (2004) and Klump *et al.* (2007b) examined the econometric estimations of production functions using data stemming from the US economy. Sato and Morita (2009) estimated the growth rates of capital and labor efficiencies for the Japanese and US economies according to equations

[1] See Klump *et al.* (2007a) for a review article on empirical studies of CES production functions.

that are derived in Sato (1970). These articles treat CES production functions with technical change that augment the efficiency of both capital and labor, and present careful analyses. Nevertheless, we consider that there is still room for improvement. For specifications for the trends in technical change, Antràs (2004) follows the bulk of the literature in assuming that the efficiencies in both labor and capital grow at constant rates.[2] Also, Klump *et al.* (2007b) assumes constant growth rates as well as a form that includes exponential, logarithmic and hyperbolic growth as special cases. It seems to give rise to inflexible models because the actual technical progress might not necessarily show the trends that Antràs (2004) and Klump *et al.* (2007b) assume. Moreover, even if the fit of the model is suitable for the relevant period, the forecast might be wide of the mark when the assumed trends are not forthcoming. For an estimation of the factor-augmenting technical changes, Sato and Morita (2009) did not *a priori* assume exponential, hyperbolic, or any other form. In that sense, the method of Sato and Morita (2009) might be superior to those of Antràs (2004) and Klump *et al.* (2007b). However, we think that problems will arise as their proposed method cannot calculate the elasticity of factor substitution directly from a CES production function with factor-augmenting technical change. To estimate the capital- and labor-augmenting technical changes, they use the elasticity of substitution derived from a CES production function with Hicks neutral technical change as a proxy.

Unlike Antràs (2004), Klump *et al.* (2007b) and Sato and Morita (2009), we address the estimation of this factor-augmenting technical change from a Bayesian perspective. Specifically, we combine the use of a Bayesian linear modeling approach in Akaike (1980) and a smoothness-priors approach in Kitagawa and Gersch (1996). By introducing the smoothness priors, we can set up the model with a sufficiently flexible structure to capture the dynamic features of the economic system, hence our model is very useful for estimating complicated behaviors of economic variables that cannot be observed directly. Studies on the application of such an approach to the analyses of dynamic structure of an economy are rare. The exceptions include Kyo and Noda (2008) and Noda and Kyo (2009a, 2009b). In Kyo and Noda (2008), smoothness priors were incorporated into a statistical model based on a Cobb-Douglas production function, and addressed, by applying Bayesian techniques, the estimation of trends in China's total factor productivity (TFP). Noda and Kyo (2009a) also incorporated a smoothness prior in a statistical model based on a Cobb-Douglas form for the production function, and estimated the trends in TFP for the Japanese prefectural economies. Based on a CES production function

[2]See, for example, Kotowitz (1968), Panik (1976), Kalt (1978), Jalava *et al.* (2006) and Masanjala and Papageorgiou (2004).

with Hicks neutral technical change, Noda and Kyo (2009b) applied the smoothness priors approach in a comparative analysis of the technical change in the Japanese and U.S. macro-economies. Recall, however, that there are some drawbacks mentioned above in the Cobb-Douglas production functions and the CES production functions with Hicks neutral technical change.

In our framework, the efficiencies of capital and labor are regarded as time-varying parameters, and thereby introduce dynamic features into capital- and labor-augmenting technical change. Parameter estimations are then carried out using the methods of maximum likelihood and Bayesian model averaging. As a practical application of the proposed Bayesian approach, we attempt to estimate the technical changes in the macro-economies of Taiwan and South Korea. The results of this analysis illustrate that our Bayesian approach can treat movements in technical change in more detail compared with the conventional production function approach, and therefore provide a useful insight into the empirical study of technical change.

The rest of the paper is organized as follows. In Section 2, we construct an analytical framework based on Bayesian models via a CES production function with factor augmenting technical change. Section 3 presents the basic scheme for parameter estimation and a procedure to apply Bayesian model averaging. In Section 4, our proposed approach is applied to time-series analysis of the macro economies in South Korea and Taiwan. Finally, Section 5 concludes the paper.

2 Bayesian Modeling based on a CES Production Function

Consider an aggregative economy in which each year a single homogeneous good is produced by two factors of production, capital and labor, under constant returns to scale technology. We follow David and van de Klundert (1965) by writing the production function

$$Q_t = \Big[(A_t K_t)^{-\rho} + (B_t L_t)^{-\rho}\Big]^{-\frac{1}{\rho}}, \tag{1}$$

where the subscript t indexes yearly increments, Q_t the value added, K_t capital, and L_t labor. Note that K_t and L_t are stocks, that is, amounts observed at one point in time, while Q_t is a flow. The model in Eq. (1) is a specification of the production function with factor-augmenting technical change.[3] The elasticity of factor substitution equals $1/(1+\rho)$, hence ρ is termed the substitution parameter. The coefficients A_t and B_t indicate the efficiency in production of capital and labor, respectively.

[3]See Sato and Beckmann (1968) for a derivation of the generalized form of such production functions.

In addition, variation in A_t with time is interpreted as capital-augmenting technical change and variation in B_t over time is interpreted as labor-augmenting technical change.

We can rewrite Eq.(1) by setting $a_t = A_t^{-\rho}$, $b_t = B_t^{-\rho}$, $x_{1t} = (-1/\rho)K_t^{-\rho}$, $x_{2t} = (-1/\rho)L_t^{-\rho}$ and $y_t = (-1/\rho)Q_t^{-\rho}$, in which notation Eq.(1) becomes

$$y_t = a_t x_{1t} + b_t x_{2t}. \tag{2}$$

Furthermore, we augment Eq. (2) to include a random disturbance:

$$y_t = a_t x_{1t} + b_t x_{2t} + \epsilon_t, \tag{3}$$

where ϵ_t is taken to be Gaussian white noise.

Noda and Kyo (2009b) applied a Bayesian approach using smoothness priors of Kitagawa and Gersch (1996) to estimate the Hicks neutral technical change. This Bayesian approach can also be applied in estimating the time-varying parameters a_t and b_t. However, it may be difficult to obtain meaningful estimates for a_t and b_t simultaneously, because there is a remarkable multicollinearity between the time-series data for x_{1t} and x_{2t}.

To ameliorate this difficulty, we simplify the model in Eq. (3) by treating b_t as a constant $\widetilde{b}$, i.e., we put $b_1 = b_2 = \cdots = \widetilde{b}$. Then, the model reduces to the following form:

$$y_t = a_t x_{1t} + \widetilde{b} x_{2t} + \epsilon_t^{(1)}, \tag{4}$$

where $\widetilde{b}$ is an unknown parameter, and $\epsilon_t^{(1)}$ is a Gaussian white noise with $\epsilon_t^{(1)} \sim \mathrm{N}(0,\ \sigma^2)$. For the model in Eq. (4), the time-varying parameter a_t is treated as a random variable from a Bayesian viewpoint. Thus, we use the following smoothness prior:

$$a_t - 2a_{t-1} + a_{t-2} = \nu_{1t}, \tag{5}$$

where ν_{1t} is a Gaussian white noise with $\nu_{1t} \sim \mathrm{N}(0, \sigma^2/d_1^2)$, and $d_1 > 0$ is an unknown parameter. It is also assumed that $\epsilon_t^{(1)}$ and ν_{1t} are independent of each other. To define a proper prior distribution for the a_t's it is necessary to set a_0 and a_{-1}. Here, all of the parameters ρ, $\widetilde{b}$, σ^2, a_0, a_{-1} and d_1 are considered as unknown constants. When the values of these parameters are given, a system of linear Bayesian equations for a_t's can be constructed based on Eqs. (4) and (5). From this, we can obtain a posterior distribution for the a_t's by using the Bayesian linear modeling method.

Alternatively, in an analogous manner, the model in Eq. (3) can also be simplified by treating a_t as constant, i.e., we assume that $a_1 = a_2 = \cdots = \widetilde{a}$. Thus, the model becomes the following form:

$$y_t = \widetilde{a}x_{1t} + b_t x_{2t} + \epsilon_t^{(2)}, \tag{6}$$

where $\widetilde{a}$ is an unknown parameter, $\epsilon_t^{(2)}$ is a Gaussian white noise with $\epsilon_t^{(2)} \sim \mathrm{N}(0,\ \sigma^2)$. Here we treat b_t as a random variable and use the smoothness prior as follows:

$$b_t - 2b_{t-1} + b_{t-2} = \nu_{2t}, \tag{7}$$

where ν_{2t} is a Gaussian white noise with $\nu_{2t} \sim \mathrm{N}(0, \sigma^2/d_2^2)$, and $d_2 > 0$ is an unknown parameter. Again, we assume that $\epsilon_t^{(2)}$ and ν_{2t} are independent of each other, and that the parameters ρ, $\widetilde{a}$, σ^2, b_0, b_{-1} and d_2 are unknown constants. When the values of these parameters are given, a system of Bayesian linear equations for the b_t's can be constructed based on Eqs. (6) and (7). Then we can obtain a posterior distribution for b_t's by using the Bayesian linear modeling method.

Thus, we have two similar Bayesian models that simplify the model of Eq. (3), specifically, those defined by the pair Eqs. (4) and (5) and the pair Eqs. (6) and (7). By combining the estimates from the two Bayesian models and using the Bayesian model averaging (BMA) approach, we can obtain synthetic estimates for the model in Eq. (3). Note that ρ and σ^2 are parameters common to the two models.

3 Parameter Estimation

3.1 Basic Scheme

We now present a procedure for estimating the time-varying parameter a_t, based on the constant-b Bayesian model, Eqs. (4) and (5). Assume that we have a data set for Q_t, K_t and L_t for a period of n points as $t = 1, 2, \ldots, n$. From Eq. (4), the likelihood of $\boldsymbol{a} = (a_1, a_2, \ldots, a_n)^{\mathtt{t}}$ and the other related parameters is given by:

$$f(\boldsymbol{Q}|\boldsymbol{a}, \rho, \widetilde{b}, \sigma^2) = J\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n \exp\left[-\frac{1}{2\sigma^2}||\boldsymbol{y} - \boldsymbol{X}\boldsymbol{a} - \widetilde{b}\boldsymbol{x}||^2\right],$$

where J is the Jacobian of the transformation from $\boldsymbol{Q}$ to $\boldsymbol{y}$ defined by $J = \Pi_{t=1}^n(\partial y_t/\partial Q_t)$. The other symbols are defined as follows:

$$\begin{aligned}
\boldsymbol{Q} &= (Q_1, Q_2, \ldots, Q_n)^{\mathtt{t}}, \\
\boldsymbol{y} &= (y_1, y_2, \ldots, y_n)^{\mathtt{t}}, \\
\boldsymbol{X} &= \mathrm{diag}(x_{11}, x_{12}, \cdots, x_{1n}), \\
\boldsymbol{x} &= (x_{21}, x_{22}, \ldots, x_{2n})^{\mathtt{t}},
\end{aligned}$$

and $|| * ||$ denotes the Euclidean norm.

When the values of σ^2, $\boldsymbol{c} = (a_{-1}, a_0)^{\mathtt{t}}$, and d_1 are given, from Eq. (5) the prior density for $\boldsymbol{a}$ is given by:

$$\pi(\boldsymbol{a}|\sigma^2, \boldsymbol{c}, d_1) = \left(\frac{d_1}{\sqrt{2\pi\sigma^2}}\right)^n \exp\left[-\frac{d_1^2}{2\sigma^2}||\boldsymbol{D}\boldsymbol{a} + \boldsymbol{B}\boldsymbol{c}||^2\right],$$

where

$$\boldsymbol{D} = \begin{bmatrix} 1 & 0 & \cdots & \cdots & \cdots & 0 \\ -2 & 1 & \ddots & & & \vdots \\ 1 & -2 & 1 & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 & -2 & 1 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}.$$

Next, we obtain the marginal likelihood of the related parameters as

$$\begin{aligned} \mathcal{L}(\rho, \widetilde{b}, \sigma^2, \boldsymbol{c}, d_1) &= \int\int\cdots\int f(\boldsymbol{Q}|\boldsymbol{a}, \rho, \widetilde{b}, \sigma^2)\pi(\boldsymbol{a}|\sigma^2, \boldsymbol{c}, d_1)d\boldsymbol{a} \\ &= J\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n d_1^n \det(\boldsymbol{W}^{\mathtt{t}}\boldsymbol{W})^{-\frac{1}{2}} \exp\left[-\frac{1}{2\sigma^2}||\boldsymbol{h} - \boldsymbol{W}\widehat{\boldsymbol{a}}||^2\right], \end{aligned} \tag{8}$$

where

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{X} \\ -d_1\boldsymbol{D} \end{bmatrix}, \quad \boldsymbol{h} = \begin{bmatrix} \boldsymbol{y} + \widetilde{b}\boldsymbol{x} \\ d_1\boldsymbol{B}\boldsymbol{c} \end{bmatrix},$$

and $\widehat{\boldsymbol{a}} = (\boldsymbol{W}^{\mathtt{t}}\boldsymbol{W})^{-1}\boldsymbol{W}^{\mathtt{t}}\boldsymbol{h}$ denotes the mean of a posterior density

$$f(\boldsymbol{a}|\boldsymbol{Q}; \rho, \widetilde{b}, \sigma^2, \boldsymbol{c}, d_1) = \frac{f(\boldsymbol{Q}|\boldsymbol{a}, \rho, \widetilde{b}, \sigma^2)\pi(\boldsymbol{a}|\sigma^2, \boldsymbol{c}, d_1)}{\int\int\cdots\int f(\boldsymbol{Q}|\boldsymbol{a}, \rho, \widetilde{b}, \sigma^2)\pi(\boldsymbol{a}|\sigma^2, \boldsymbol{c}, d_1)d\boldsymbol{a}}, \tag{9}$$

for $\boldsymbol{a}$. Therefore, the estimates, $\widehat{\rho}$, $\widehat{\widetilde{b}}$, $\widehat{\sigma}^2$, $\widehat{\boldsymbol{c}}$, and $\widehat{d_1}$ for the related parameters can be obtained by maximizing the likelihood $\mathcal{L}(\rho, \widetilde{b}, \sigma^2, \boldsymbol{c}, d_1)$ in Eq. (8).

Computationally, if the values of ρ and d_1 are given, then $\boldsymbol{a}$ together with $\boldsymbol{c}$ and $\widetilde{b}$ can be estimated simultaneously by using the least squares method (see for example, Jiang, 1995). That is, when we put

$$\boldsymbol{z} = \begin{bmatrix} \boldsymbol{y} \\ \boldsymbol{0}_n \end{bmatrix}, \quad \boldsymbol{V} = \begin{bmatrix} \boldsymbol{X} & \boldsymbol{O} & \boldsymbol{x} \\ d_1\boldsymbol{D} & d_1\boldsymbol{B} & \boldsymbol{0}_n \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{c} \\ \widetilde{b} \end{bmatrix}$$

with $\boldsymbol{O}$ denoting a zero matrix, and $\boldsymbol{0}_n$ being an n-dimensional vector of zeros, the estimate,

$$\begin{aligned} \widehat{\boldsymbol{\beta}}(\rho, d_1) &= \left(\widehat{\boldsymbol{a}}^{\mathtt{t}}(\rho, d_1), \widehat{\boldsymbol{c}}^{\mathtt{t}}(\rho, d_1), \widehat{\widetilde{b}}(\rho, d_1)\right)^{\mathtt{t}} \\ &= (\boldsymbol{V}^{\mathtt{t}}\boldsymbol{V})^{-1}\boldsymbol{V}^{\mathtt{t}}\boldsymbol{z}, \end{aligned}$$

for $\boldsymbol{\beta}$ is obtained. Thus, by substituting $\widetilde{b} = \widehat{\widetilde{b}}(\rho, d_1)$ and $\boldsymbol{c} = \widehat{\boldsymbol{c}}(\rho, d_1)$ into $\mathcal{L}(\rho, \widetilde{b}, \sigma^2, \boldsymbol{c}, d_1)$ in Eq. (8), the partial likelihood of ρ, σ^2 and d_1 can be obtained as

$$\mathcal{L}_1(\rho, \sigma^2, d_1) = \mathcal{L}(\rho, \widehat{\widetilde{b}}, \sigma^2, \widehat{\boldsymbol{c}}, d_1). \tag{10}$$

Correspondingly, a conditional posterior density, $g(\boldsymbol{a}|\boldsymbol{Q}; \rho, \sigma^2, d_1)$, for $\boldsymbol{a}$ can be derived from $f(\boldsymbol{a}|\boldsymbol{Q}; \rho, \widetilde{b}, \sigma^2, \boldsymbol{c}, d_1)$ in Eq. (9) by:

$$g(\boldsymbol{a}|\boldsymbol{Q}; \rho, \sigma^2, d_1) = f\Big(\boldsymbol{a}|\boldsymbol{Q}; \rho, \widehat{\widetilde{b}}(\rho, d_1), \sigma^2, \widehat{\boldsymbol{c}}(\rho, d_1), d_1\Big).$$

Moreover, the estimate $\widehat{d_1}$ of the parameter d_1 is obtained by maximizing the marginal likelihood $\mathcal{L}_1(\rho, \sigma^2, d_1)$ in Eq. (10) for given ρ and σ^2. Thus, we obtain the partial likelihood of ρ and σ^2 as

$$\mathcal{L}_1^*(\rho, \sigma^2) = \mathcal{L}_1(\rho, \sigma^2, \widehat{d_1}). \tag{11}$$

Note that the procedure for estimating the parameter $\boldsymbol{b} = (b_1, b_2, \ldots, b_n)^{\mathsf{t}}$ in the constant-a Bayesian model, Eqs. (6) and (7), is the same, and is omitted here for brevity. For this model, we can obtain the partial likelihood $\mathcal{L}_2(\rho, \sigma^2, d_2)$ of ρ, σ^2 and d_2 by a procedure similar to the above basic scheme. Then the partial likelihood of ρ and σ^2 can be obtained by using the maximum likelihood estimate $\widehat{d_2}$ of d_2 as

$$\mathcal{L}_2^*(\rho, \sigma^2) = \mathcal{L}_2(\rho, \sigma^2, \widehat{d_2}). \tag{12}$$

3.2 Bayesian Model Averaging Approach

To obtain the estimates for ρ and σ^2 and the final estimates for parameters $\boldsymbol{a}$ and $\boldsymbol{b}$ of the model in Eq. (3), we use the BMA approach as follows. Here, we reconsider the two Bayesian models with constant-b and constant-a; these models are identified here as M_1 and M_2 respectively. ¿From the BMA perspective (see for example, Claeskens and Hjort, 2008), it is assumed that all models $\{\mathrm{M}_1, \mathrm{M}_2\}$ are uncertain and all have equal uncertainties. Thus, we use a uniform prior for the models $\{\mathrm{M}_1, \mathrm{M}_2\}$ as follows:

$$q(\mathrm{M}_1) = q(\mathrm{M}_2) = \frac{1}{2},$$

where $q(\mathrm{M}_i)$ is the prior probability for the i-th model M_i $(i = 1, 2)$. As shown in the preceding subsection, a partial likelihood for the model M_i is $\mathcal{L}_i^*(\rho, \sigma^2)$, as given in Eqs. (11) and (12). Therefore, a conditional posterior probability for model M_i is given by

$$\begin{aligned} p(\mathrm{M}_i|\boldsymbol{Q}; \rho, \sigma^2) &= \frac{\mathcal{L}_i^*(\rho, \sigma^2) q(\mathrm{M}_i)}{\mathcal{L}_1^*(\rho, \sigma^2) q(\mathrm{M}_1) + \mathcal{L}_2^*(\rho, \sigma^2) q(\mathrm{M}_2)} \\ &= \frac{\mathcal{L}_i^*(\rho, \sigma^2)}{\mathcal{L}_1^*(\rho, \sigma^2) + \mathcal{L}_2^*(\rho, \sigma^2)}. \end{aligned}$$

Moreover, from the basic estimation scheme just mentioned, we can see that for model M_1 given ρ and d_1, the posterior mean of $\boldsymbol{a}$ is $\widehat{\boldsymbol{a}}(\rho, d_1)$ and the estimate of $\boldsymbol{b} = \widetilde{b}\mathbf{1}_n$ is $\widehat{\widetilde{b}}(\rho, d_1)\mathbf{1}_n$. Similarly, for model M_2 given ρ and d_2, the posterior mean of $\boldsymbol{b}$ is $\widehat{\boldsymbol{b}}(\rho, d_2)$ and the estimate of $\boldsymbol{a} = \widetilde{a}\mathbf{1}_n$ is $\widehat{\widetilde{a}}(\rho, d_2)\mathbf{1}_n$. Therefore, based on the Bayesian model averaging approach, the synthetic estimates for $\boldsymbol{a}$ and $\boldsymbol{b}$ are given by (see, Draper, 1995):

$$\begin{aligned}\widehat{\widehat{\boldsymbol{a}}}(\rho, \sigma^2) &= p(\mathrm{M}_1|\boldsymbol{Q}; \rho, \sigma^2)\widehat{\boldsymbol{a}}(\rho, \widehat{d}_1) + p(\mathrm{M}_2|\boldsymbol{Q}; \rho, \sigma^2)\widehat{\widetilde{a}}(\rho, \widehat{d}_2)\mathbf{1}_n, \\ \widehat{\widehat{\boldsymbol{b}}}(\rho, \sigma^2) &= p(\mathrm{M}_2|\boldsymbol{Q}; \rho, \sigma^2)\widehat{\boldsymbol{b}}(\rho, \widehat{d}_2) + p(\mathrm{M}_1|\boldsymbol{Q}; \rho, \sigma^2)\widehat{\widetilde{b}}(\rho, \widehat{d}_1)\mathbf{1}_n.\end{aligned}$$

Furthermore, we consider the quantity

$$\begin{aligned}\overline{\mathcal{L}}(\rho, \sigma^2) &= \mathcal{L}_1^*(\rho, \sigma^2)q(\mathrm{M}_1) + \mathcal{L}_2^*(\rho, \sigma^2)q(\mathrm{M}_2) \\ &= \frac{1}{2}\Big(\mathcal{L}_1^*(\rho, \sigma^2) + \mathcal{L}_2^*(\rho, \sigma^2)\Big) \qquad (13)\end{aligned}$$

to be a measure of the fitting reliability for the model in Eq. (3) with respect to the parameters ρ and σ^2. Thus, the estimates $\widehat{\rho}$ and $\widehat{\sigma}^2$ of ρ and σ^2 can be obtained by maximizing the mean likelihood $\overline{\mathcal{L}}(\rho, \sigma^2)$ in Eq. (13) with respect to these parameters. Hence, the final estimates for $\boldsymbol{a}$ and $\boldsymbol{b}$ are obtained as

$$\widehat{\widehat{\boldsymbol{a}}} = \widehat{\widehat{\boldsymbol{a}}}(\widehat{\rho}, \widehat{\sigma}^2), \quad \widehat{\widehat{\boldsymbol{b}}} = \widehat{\widehat{\boldsymbol{b}}}(\widehat{\rho}, \widehat{\sigma}^2).$$

4 Application

4.1 Data Sources

To estimate the CES production function for Taiwan, we use real GDP (Q_t) and non-land fixed physical capital (K_t) from Chow and Lin (2002).[4] In addition, labor data (L_t) are calculated according to the product of the number of employed workers from Mizoguchi (2008) and the quality of human capital per worker from Chow and Lin (2002). The annual time-series data for Taiwan cover the period from 1951 to 1999.

Also, we conducted parameter estimations of the CES production function for South Korea using macro-data taken from Pyo (2001), which are real GDP (Q_t), physical capital (K_t), and the number of employed workers (L_t). The annual time-series data for South Korea covers a similar period, from 1946 to 1999.

[4]The non-land fixed capital comprises fixed assets excluding land values. Chow and Lin (2002) excluded land values and cumulative depreciation from total gross fixed assets to obtain a net value for non-land fixed assets.

4.2 Results and Discussion

When we calculated the estimation procedure presented in Section 3, we used yearly data with values from each year divided by the value in the initial year, that is, Q_t/Q_1, K_t/K_1 and L_t/L_1 for $t = 1, 2, \ldots, n$, where $t = 1$ denotes the initial year and $t = n$ denotes the final year in our sample. Considering this data, we follow de La Grandville (2009) by rewriting Eq. (1) as

$$\frac{Q_t}{Q_1} = \left[\delta\left(h_t\frac{K_t}{K_1}\right)^{-\rho} + (1-\delta)\left(g_t\frac{L_t}{L_1}\right)^{-\rho}\right]^{-\frac{1}{\rho}}. \tag{14}$$

In Eq. (14), we call h_t and g_t the modified efficiencies in production of capital and labor, respectively. Under the assumptions $h_1 = g_1 = 1$ and competitive factor imputation, δ denotes the share of capital income from the total income at base year $t = 1$ and $(1-\delta)$ denotes the corresponding share of labor income (see, de La Grandville, 2009 for details). Note that we have relations $a_t = A_t^{-\rho} = \delta h_t^{-\rho}$ and $b_t = B_t^{-\rho} = (1-\delta)g_t^{-\rho}$.

As mentioned earlier, the CES production function with Hicks neutral technical change is a special case of the CES production function with factor augmenting technical change. To confirm the superiority of the factor augmenting type as a statistical model, we estimated not only the CES production function with factor augmenting technical change in the present paper, but also the CES production function with Hicks neutral technical change, such as Noda and Kyo (2009b) using the data set mentioned above. As a result, when we use the Taiwan data, the value of the Akaike information criterion (AIC), given Hicks neutral type is -17.052, while that for the factor augmenting type is -398.025. Also, when we use the South Korea data, the value of AIC, given Hicks neutral type is -0.017, while that for the factor augmenting type is -367.431. Consequently, from the viewpoint of the minimum AIC model selection procedure, it is obvious that the factor augmenting type is better model. Thus, we present only the results of the estimation for the CES production function with factor augmenting technical change.

For Taiwan, the estimated results of the constant parameters are as follows: $\widehat{d}_1 = 3.4924$, $\widehat{d}_2 = 4.4152$, $\widehat{\rho} = -0.09418$, and $\widehat{\sigma}^2 = 4.0896 \times 10^{-5}$. For South Korea, these are: $\widehat{d}_1 = 10.7064$, $\widehat{d}_2 = 14.3917$, $\widehat{\rho} = -0.1504$, and $\widehat{\sigma}^2 = 4.6191 \times 10^{-4}$. The estimate for the elasticity of substitution can be calculated from the estimate $\widehat{\rho}$. Specifically, these estimates for Taiwan and South Korea are 1.104 and 1.177, respectively. De La Grandville (2009) derived and proved two remarkable theorems on the relationship between the elasticity of substitution and performance of macroeconomy. These can be stated as follows:

(i) *If an economy is driven by a CES production function exhibiting capital- and labor-augmenting technical progress, then at any stage of its development per capita income is an increasing function of its elasticity of substitution.*

(ii) *If an economy is driven by a CES production function exhibiting capital- and labor-augmenting technical progress, then at any stage of its development the cost of producing one unit of national product is a decreasing function of the elasticity of substitution.*

According to these two theorems, we can conjecture that the long-term per capita income in South Korea may be greater than that in Taiwan, and the long-term cost of producing one unit national product in South Korea may be lower than in Taiwan.

Tables 1 and 2 show the estimates $\widehat{a}_t$ and $\widehat{b}_t$ for Taiwan and South Korea respectively.

Table 1: Estimated results of the time-varying parameters for Taiwan

	$\widehat{a}_t$		$\widehat{a}_t$		$\widehat{b}_t$		$\widehat{b}_t$
1951	0.69462	1976	0.72444	1951	0.86421	1976	0.97354
1952	0.69677	1977	0.72551	1952	0.87066	1977	0.97838
1953	0.69865	1978	0.72671	1953	0.87635	1978	0.98364
1954	0.70021	1979	0.72757	1954	0.88116	1979	0.98788
1955	0.70140	1980	0.72829	1955	0.88486	1980	0.99168
1956	0.70237	1981	0.72884	1956	0.88795	1981	0.99478
1957	0.70324	1982	0.72918	1957	0.89080	1982	0.99699
1958	0.70405	1983	0.72984	1958	0.89344	1983	1.00015
1959	0.70502	1984	0.73075	1959	0.89670	1984	1.00422
1960	0.70609	1985	0.73148	1960	0.90033	1985	1.00776
1961	0.70730	1986	0.73257	1961	0.90451	1986	1.01254
1962	0.70868	1987	0.73384	1962	0.90933	1987	1.01817
1963	0.71029	1988	0.73490	1963	0.91501	1988	1.02337
1964	0.71209	1989	0.73579	1964	0.92147	1989	1.02806
1965	0.71363	1990	0.73652	1965	0.92710	1990	1.03222
1966	0.71484	1991	0.73730	1966	0.93169	1991	1.03647
1967	0.71581	1992	0.73801	1967	0.93555	1992	1.04056
1968	0.71665	1993	0.73871	1968	0.93907	1993	1.04472
1969	0.71763	1994	0.73939	1969	0.94313	1994	1.04880
1970	0.71891	1995	0.74008	1970	0.94839	1995	1.05300
1971	0.72044	1996	0.74081	1971	0.95466	1996	1.05735
1972	0.72178	1997	0.74145	1972	0.96023	1997	1.06128
1973	0.72244	1998	0.74187	1973	0.96329	1998	1.06438
1974	0.72249	1999	0.74237	1974	0.96442	1999	1.06763
1975	0.72311			1975	0.96774		

Table 2: Estimated results of the time-varying parameters for South Korea

	$\widehat{a}_t$		$\widehat{a}_t$		$\widehat{b}_t$		$\widehat{b}_t$
1946	0.49844	1973	0.48799	1946	0.54203	1973	0.53809
1947	0.49712	1974	0.48837	1947	0.54107	1974	0.53880
1948	0.49577	1975	0.48861	1948	0.54007	1975	0.53942
1949	0.49414	1976	0.48860	1949	0.53876	1976	0.53986
1950	0.49235	1977	0.48810	1950	0.53721	1977	0.53991
1951	0.49099	1978	0.48699	1951	0.53604	1978	0.53943
1952	0.49024	1979	0.48552	1952	0.53545	1979	0.53863
1953	0.49018	1980	0.48420	1953	0.53556	1980	0.53793
1954	0.49019	1981	0.48389	1954	0.53577	1981	0.53810
1955	0.48990	1982	0.48486	1955	0.53573	1982	0.53940
1956	0.48915	1983	0.48686	1956	0.53529	1983	0.54167
1957	0.48792	1984	0.48930	1957	0.53445	1984	0.54440
1958	0.48625	1985	0.49176	1958	0.53321	1985	0.54722
1959	0.48428	1986	0.49419	1959	0.53166	1986	0.55005
1960	0.48235	1987	0.49632	1960	0.53010	1987	0.55263
1961	0.48095	1988	0.49792	1961	0.52904	1988	0.55478
1962	0.48029	1989	0.49882	1962	0.52864	1989	0.55636
1963	0.48040	1990	0.49929	1963	0.52894	1990	0.55757
1964	0.48106	1991	0.49951	1964	0.52971	1991	0.55854
1965	0.48189	1992	0.49960	1965	0.53063	1991	0.55937
1966	0.48280	1993	0.49983	1966	0.53161	1993	0.56021
1967	0.48366	1994	0.50021	1967	0.53256	1994	0.56109
1968	0.48457	1995	0.50045	1968	0.53360	1995	0.56175
1969	0.48552	1996	0.50020	1969	0.53471	1996	0.56192
1970	0.48632	1997	0.49947	1970	0.53570	1997	0.56164
1971	0.48697	1998	0.49894	1971	0.53656	1998	0.56145
1972	0.48748	1999	0.49917	1972	0.53731	1999	0.56177

Because, by definition, $a_t = A_t^{-\rho} = \delta h_t^{-\rho}$ and $b_t = B_t^{-\rho} = (1-\delta)g_t^{-\rho}$, we obtain $a_t^{-1/\rho} = A_t = \delta^{-1/\rho} h_t$ and $b_t^{-1/\rho} = B_t = (1-\delta)^{1/\rho} g_t$. Furthermore, the relations $A_t/A_1 = h_t/h_1$ and $B_t/B_1 = g_t/g_1$ hold. In other words, the index of efficiency of capital (A_t/A_1) synchronizes with the index of the modified efficiency of capital (h_t/h_1). A similar remark holds between (B_t/B_1) and (g_t/g_1). We can obtain the estimates $\widehat{A}_t$ and $\widehat{B}_t$ from $\widehat{a}_t^{-1/\widehat{\rho}}$ and $\widehat{b}_t^{-1/\widehat{\rho}}$, respectively.

Figure 1 presents graphs for $(\widehat{A}_t/\widehat{A}_1) \times 100$ and $(\widehat{B}_t/\widehat{B}_1) \times 100$ for Taiwan.

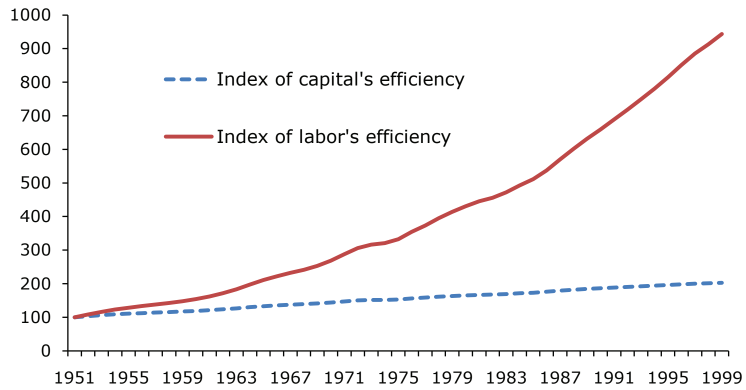

Figure 1: The indices of capital efficiency and labor efficiency for Taiwan

From Figure 1, we find clearly that the efficiencies of labor and capital in Taiwan have changed at different growth rates. In particular, the rapid growth in labor efficiency is remarkable. This result suggests that labor-augmenting technical progress had contributed much to Taiwan's economic growth during the five-decade period.

Figure 2 shows the graphs of $(\widehat{A}_t/\widehat{A}_1) \times 100$ and $(\widehat{B}_t/\widehat{B}_1) \times 100$ for South Korea.

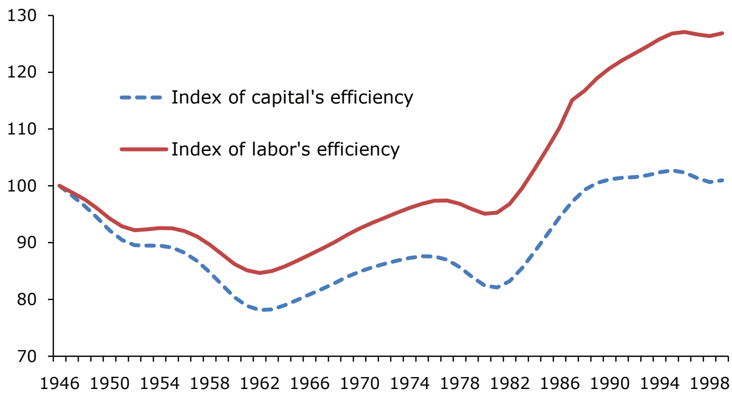

Figure 2: The indices of capital efficiency and labor efficiency for South Korea

The following features are seen from Figure 2. First, movements in labor and capital efficiencies for South Korea are more complex than those of Taiwan. Moreover, we find a similar pattern between the efficiencies, although the curves differ in magnitude. An upward tendency is seen in both over the period from the 1980s to the early 1990s. However, this movement stagnated during the late 1990s.

The following conclusions are obtained from a comprehensive viewpoint. First, there is a possibility that capital efficiency and labor efficiency have distinctly different trends. Figure 1 for Taiwan is one such example. Moreover, as shown in Figure 2 for South Korea, movements of efficiencies of capital and labor may be non-monotonic. Thus, our practical application suggests that modeling with a high degree of flexibility is indispensable in capturing the trends in technical change. In other words, the model should be able to deal with the diversity and complexity in the behavior of technical change. However, many earlier studies impose stringent constraints on the trends in efficiencies of capital and labor, such as the efficiencies being exponentially dependent on time. Thus, conventional approaches that lack the necessary flexibility within the model cannot estimate trends in technical change precisely, and might be misleading.

5 Conclusion

Recently, the underlying role of technical progress in economic growth has become increasingly important. However, statistical methods for estimating technical change have so far not been sufficiently developed. We improve the conventional production function based approaches by using Bayesian modeling, and propose a new methodology in the time-series analysis of capital- and labor-augmenting technical change.

As an illustration, we applied our proposed approach to an analysis of data obtained for Taiwan and South Korea. Our main results can be summarized as follows: The Taiwanese economy experienced rapid growth in labor efficiency from 1950s to 1990s, while capital efficiency showed a modest growth during the same period. This implies that the labor-augmenting technical progress had contributed much to Taiwan's economic growth over the period. For South Korea, a similar pattern was seen for labor and capital efficiencies, although each developed at different levels and growth rates. In particular, both had shown an upward tendency from the 1980s to the early 1990s, but movements stagnated during the late 1990s. Our estimations suggest that to sustain future economic growth in South Korea, policies need to be established to promote both capital- and labor-augmenting technical progress.

To our knowledge, attempts at analyzing concepts like technical progress as in the present study have not yet been reported. The Bayesian perspective as contributed here sheds light on an approach to capture such concepts as the factor augmenting technical change. In particular, because our approach enables a statistical analysis of trends in technical change in a rigorous manner by a flexible structure of the models introducing smoothness priors, it has definite superiority over conventional approaches. Therefore, our approach can be regarded as a promising tool for an empirical study of technical change.

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